COSMOLOGY / introductory remarks

- very old science in human history
- practical and speculative side

calendar, ephemeris

magic/mythological ideas about origin world

Hindus: $T_{universe}$ = 1 Brahma day = 4.32×10^9 yr

- very rapid evolution since ~1960
  speculative backyard → quantitative science

- CONTENTS OF COURSE

  RELATIVITY & GEOMETRY

  EVOLUTION OUR FRW UNIVERSE

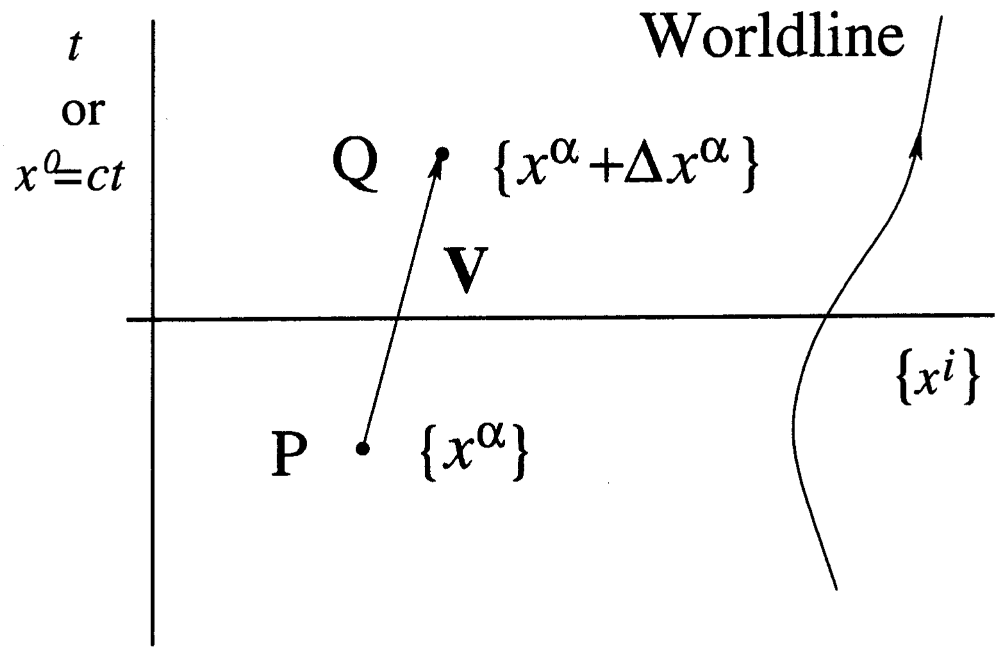
  BIG BANG PHYSICS

  OBSERVATIONAL COSMOLOGY

  INFLATION

will have to skip many issues!

SPECIAL RELATIVITY



METRIC

Distance between P and Q:

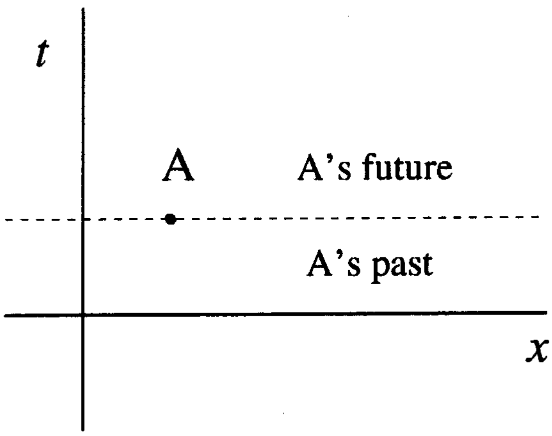
$$\Delta s^2 = \underbrace{(\Delta x^0)^2}_{(c\,dt)^2} - \Delta x^i \Delta x^i = \eta_{\alpha\beta}\, \Delta x^\alpha \Delta x^\beta$$

$$\eta = \begin{pmatrix} +1 & & & \emptyset \\ & -1 & & \\ & & -1 & \\ \emptyset & & & -1 \end{pmatrix}$$ note sign convention

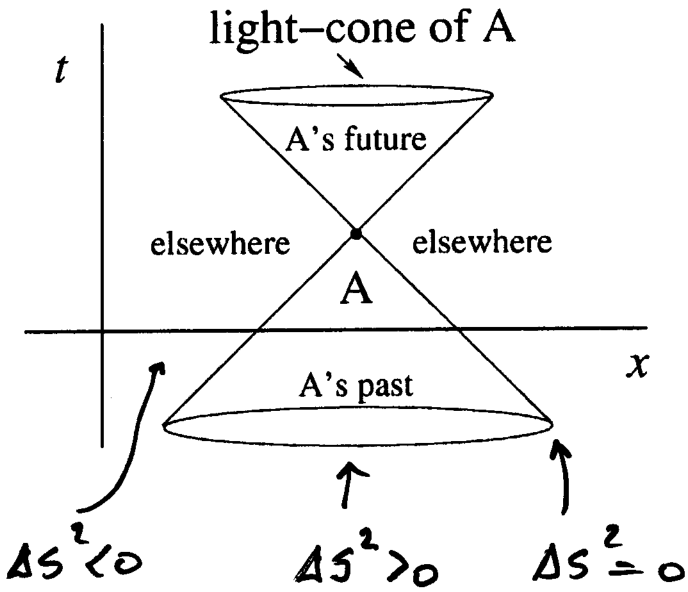
- notation: $\Delta s^2 \equiv (\Delta s)^2$; $dt^2 \equiv (dt)^2$ etc
- Δs^2 is invariant

SPECIAL RELATIVITY

Before SR

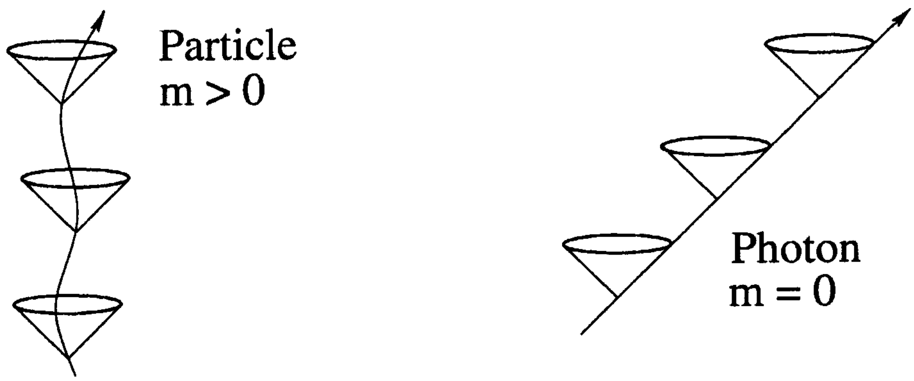


After SR



- Δs^2 invariant $\longrightarrow$ all observers come to the same result

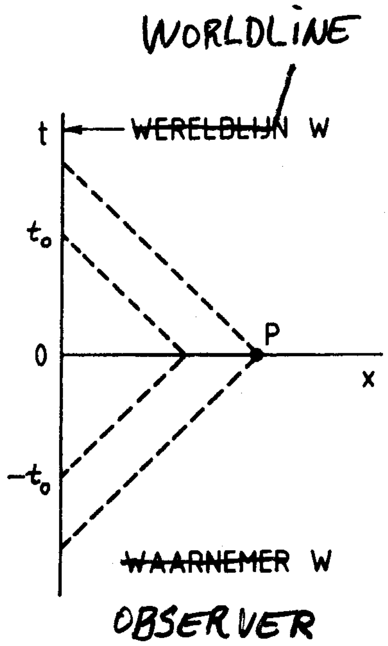
  we can speak of "the" lightcone



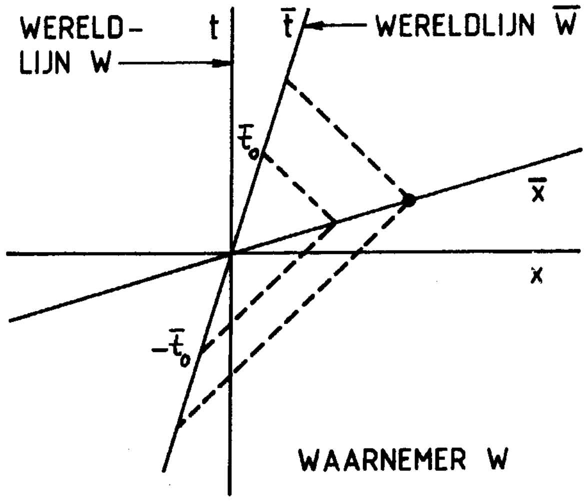
$P\ \{x^\alpha\}$ & $Q\ \{x^\alpha + \Delta x^\alpha\}$

$$\Delta s^2 = \eta_{\alpha\beta}\, \Delta x^\alpha\, \Delta x^\beta$$

$\Delta s^2 > 0$	TIMELIKE	(MATTER)
$= 0$	NULL VECTOR	(PHOTONS)
< 0	SPACELIKE	(TACHYONS)

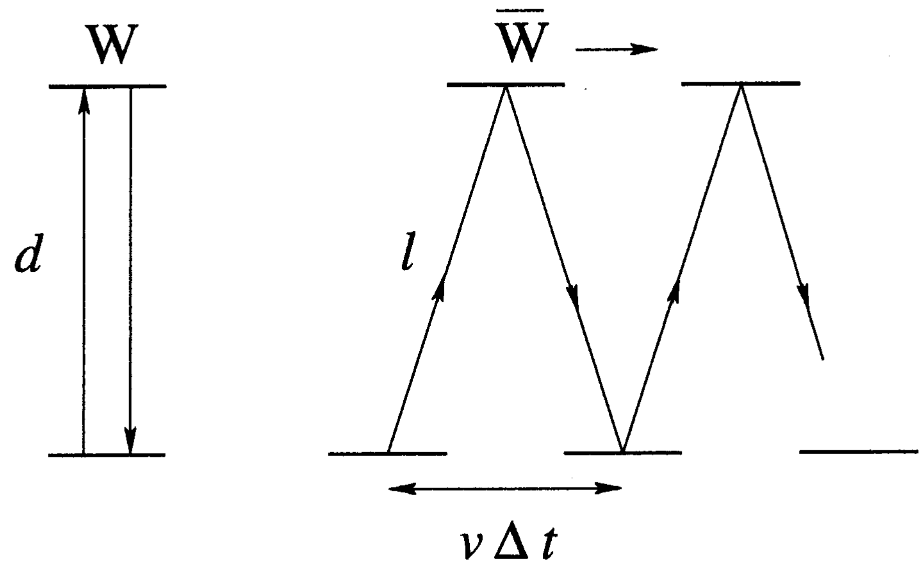


WORLDLINE
t
~~WERELDLIJN~~ W
t_0
0
P
x
$-t_0$
~~WAARNEMER~~ W
OBSERVER



WERELD-
LIJN W
t
$\bar{t}$
WERELDLIJN $\overline{W}$
$\bar{t}_0$
$\bar{x}$
x
$-\bar{t}_0$
WAARNEMER W

EINSTEIN CLOCK



- proper time
- Lorentz transformations
- tensors

GENERAL RELATIVITY

- arbitrarily moving frames $\longrightarrow$ forces closely related to gravity

- classical gravity $\quad \nabla^2\phi = 4\pi G\rho \;;\; \underline{F} = -m\underline{\nabla}\phi$

  holds only in one frame

  $\rightarrow \nabla^2 \longrightarrow \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$??

  $\rightarrow$ SR-like theories with one global frame seem dead end.

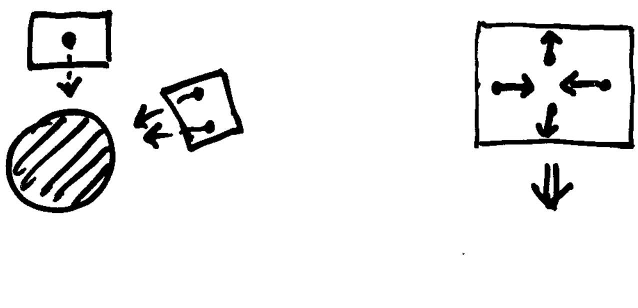
- weak equivalence

$$m_i \ddot{\underline{r}} = \text{applied force}$$
$$= \text{-----} - m_g \underline{\nabla}\phi$$

m_i / m_g same for all bodies, say $\equiv 1$

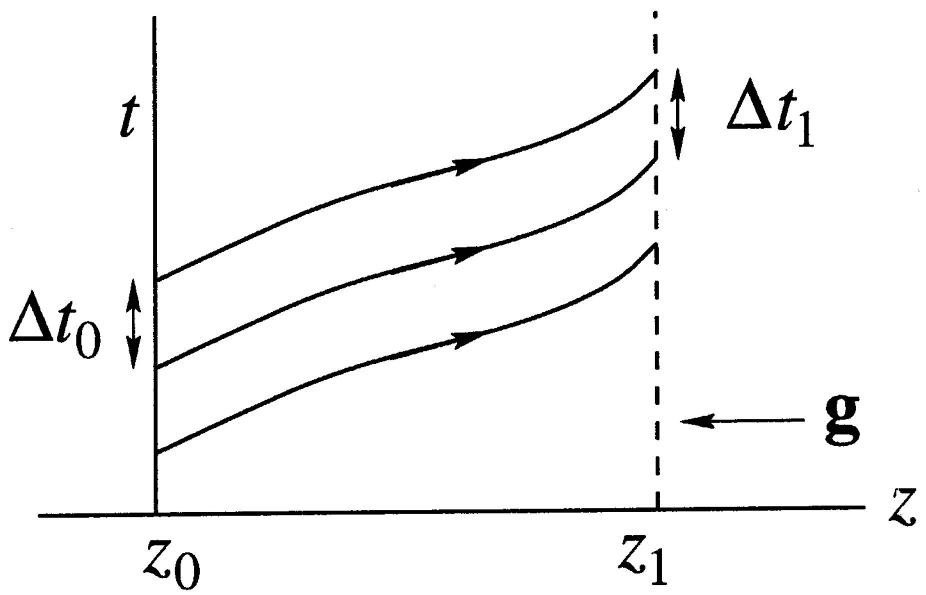
(Eötvös 10^{-8}, now $\sim 10^{-12}$)

$\therefore$ gravity can be transformed away locally not globally.



Tidal forces cannot be transformed away

Real gravity field is inhomogeneous



- $h = z_1 - z_2 = 22.5\,m$!

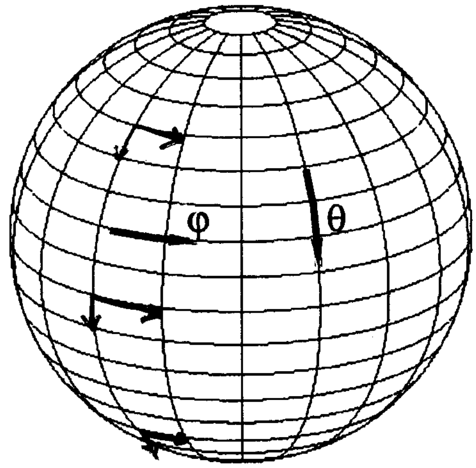
  (Pound, Rebka & Snider 1961, 1965)

- Experiment: $\Delta t_1 > \Delta t_0$ (redshift)

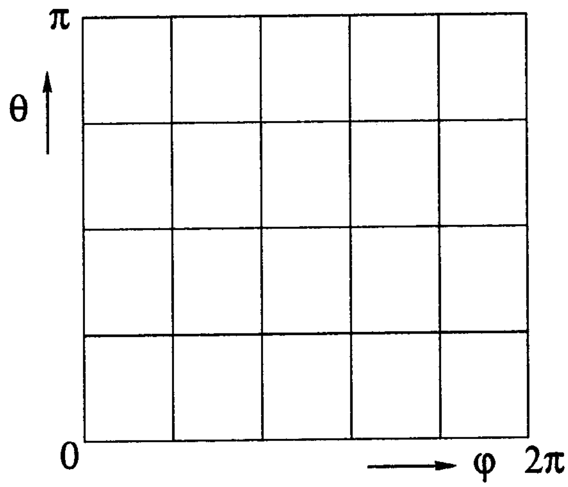
- suggests that spacetime is curved due to gravity

- $\Delta s^2 = \eta_{\alpha\beta}\, \Delta x^\alpha \Delta x^\beta \rightarrow ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$

  metric tensor determined by mass distribution



Geometrical picture



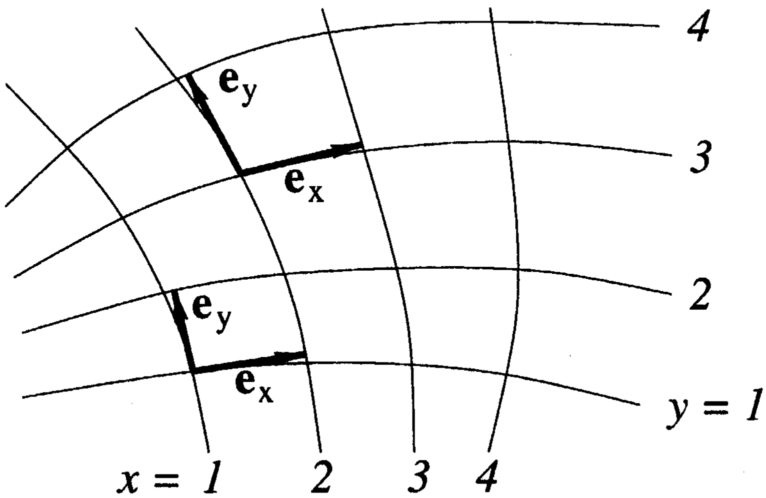
Co–ordinate picture

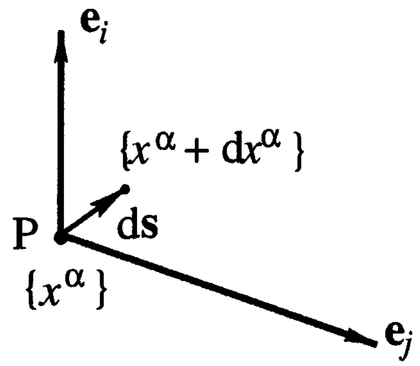
<u>metric</u>:

$$ds^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$$

$\underline{e}_\theta \cdot \underline{e}_\theta$ $\quad$ $\underline{e}_\varphi \cdot \underline{e}_\varphi$

co-ordinate lines ; tangent space





- Co-ordinate lines : do as you like
- base vectors : tangent to co-ord. lines, in + direction
- base vector span flat tangent space

  (presupposes existence flat embedding space)

- Metric in tangent space : arbitrary, but there is one preferred metric which is very handy

  infinitesimal vector $d\underline{s} = dx^\alpha \underline{e}_\alpha$ has length ds of Riemann space

  $$\Rightarrow \quad \boxed{g_{\alpha\beta} = \underline{e}_\alpha \cdot \underline{e}_\beta}$$

Contra- & Covariant

- Finite vectors in tangent space $\quad \underline{A} = A^\alpha \underline{e}_\alpha$
- Nb. all vectors associated with a particle lie in the local tangent space ($\underline{v}$, $\underline{a}$, spin ...)
- Definition of A_α :

$$\underline{A}\cdot\underline{A} = A^\alpha \underline{\underline{A_\alpha}} \quad \text{(summation)}$$

$\left.\begin{array}{l}\text{covariant}\\ \text{contravariant}\end{array}\right\}$ components of $\underline{A}$

$$= \underline{\underline{g_{\alpha\beta}}}\, A^\alpha \underline{\underline{A}}^\beta \qquad (\underline{A}\cdot\underline{A} = \text{length!})$$

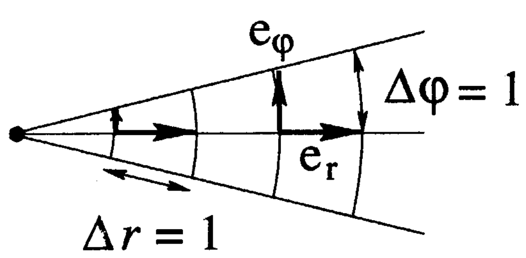
$\therefore \quad A_\alpha = g_{\alpha\beta} A^\beta \qquad$ Index lowering

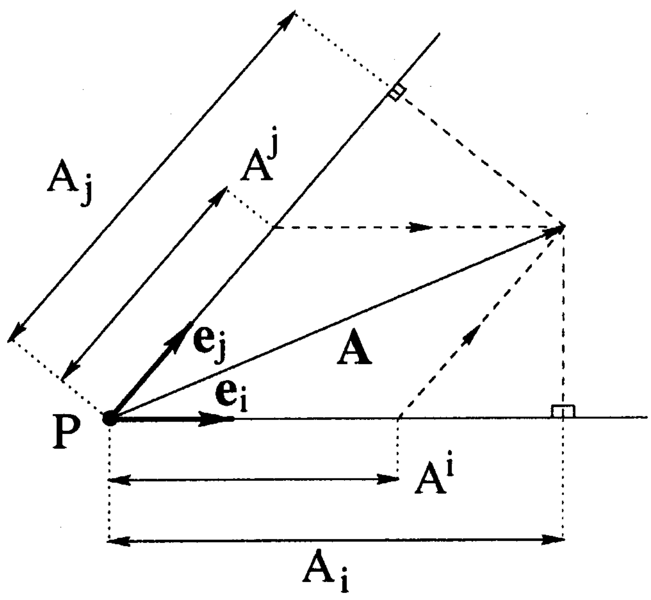
- Index raising: $\quad A^\alpha \underset{D}{=} g^{\alpha\gamma} A_\gamma$

$$= g^{\alpha\gamma} g_{\gamma\nu} A^\nu$$

$$\therefore \quad g^{\alpha\gamma} g_{\gamma\nu} = \delta^\alpha_\nu = \begin{cases} 1 & \alpha = \nu \\ 0 & \alpha \neq \nu \end{cases}$$

or: $\{g^{\alpha\gamma}\}$ is inverse of $\{g_{\gamma\nu}\}$





Interpretation A^α and A_α

$\underline{A} = A^\alpha \underline{e}_\alpha$ $\longrightarrow$ A^α components $\underline{A}$ along basis (parallelogram construction)

$$A_\gamma = g_{\gamma\alpha} A^\alpha = \underline{e}_\gamma \cdot \underline{e}_\alpha A^\alpha = \underline{e}_\gamma \cdot \underline{A}$$

$\rightarrow$ A_α is projection $\underline{A}$ on base-vector $\underline{e}_\alpha$

- All this holds for **any** vector field $\underline{A}(x^\alpha) \begin{smallmatrix} \nearrow A^\alpha \\ \searrow A_\alpha \end{smallmatrix}$

- <u>TENSORS?</u> <u>Behaviour under coordinate transf.</u>

$$\{x^\alpha\} \;\&\; \{\bar{x}^\beta\} \quad \rightarrow \quad \delta \bar{x}^\beta = \frac{\partial \bar{x}^\beta}{\partial x^\alpha}\, \delta x^\alpha$$

<u>DEF</u>: Any set of numbers that transforms in this way is a (contravariant) tensor of 1st rank

i.e.
$$\bar{A}^\nu = \frac{\partial \bar{x}^\nu}{\partial x^\alpha}\, A^\alpha \qquad \text{Nb } L^\nu_{\ \alpha}$$

- <u>Tensor of higher rank</u>

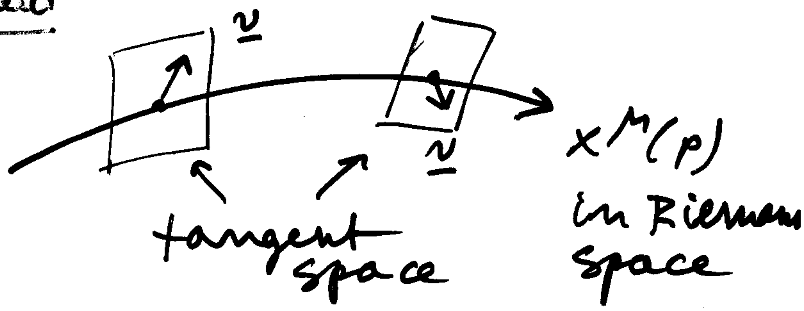
$$T^{\mu\nu} = A^\mu B^\nu \quad ; \quad \varphi^{\alpha\ \gamma}_{\ \beta} = A^\alpha B_\beta C^\gamma \quad \text{etc}$$

each index transforms according to

$$\bar{T}^{\mu\nu} = \frac{\partial \bar{x}^\mu}{\partial x^\alpha} \frac{\partial \bar{x}^\nu}{\partial x^\beta}\, T^{\alpha\beta} \qquad \text{etc}$$

$g_{\mu\nu}$ is basic ingredient

- <u>Indexgymnastics</u> $\quad T_{\mu\nu} = g_{\mu\alpha}\, g_{\nu\beta}\, T^{\alpha\beta}$ etc
- <u>Parallel displacement</u>



- <u>geodesics</u> (orbits of test particles)
- <u>curvature</u>

FIELD EQUATION?

- $\nabla^2 \phi = 4\pi G \rho_0 \Rightarrow$ what??

- CURVATURE OF SPACE TIME (:) TOTAL ENERGY DENSITY

- $$\boxed{G^{\mu\nu}(\{g_{\alpha\beta}\}) = -\frac{8\pi G}{c^2} \cdot T^{\mu\nu}}$$

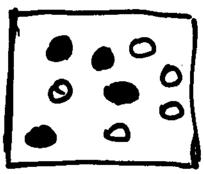
  by considering weak fields → classical mechanics

  Rest energy density
  pressure
  EM fields ---

- Simplest $T^{\mu\nu} = \rho_0 u^\mu u^\nu$ "DUST"

  4-velocity $\frac{1}{c}\frac{dx^\mu}{d\tau} \simeq (\gamma, v_i/c)$

- Why relation between <u>tensors of 2nd rank</u>?



$$m_0 \to m = \gamma m_0$$

$$M_0 \to M = \gamma M_0$$

$$\therefore \rho_0 = m_0 M_0 \to \boxed{\rho = mM = \gamma^2 \rho_0}$$

$\Rightarrow$ 0,0 component of <u>2nd</u> rank tensor.

- Matter tells spacetime how to curve ($g_{\alpha\beta}$)
  spacetime tells matter how to move (along geodesics determined by $g_{\alpha\beta}$)

GENERAL RELATIVITY / RECAPITULATION

- WEAK EQUIVALENCE — gravity is partly an apparent force

- ORBIT OF TEST PARTICLE IS GEODESIC — "straight" orbit in curved spacetime. Curvature due to Σ energy densities

- WHY CURVATURE?

- RIEMANN SPACES — tangent space, finite vectors, contra- & covariant components. Tensors : transformations.

- $\nabla^2 \phi = 4\pi G \rho \Rightarrow$??

ρ transforms as 0,0 component of second rank tensor

$$\rho \rightarrow \rho u^\mu u^\nu$$

$$u^\mu = \frac{1}{c}\left(\frac{dx^0}{d\tau}, \frac{dx^i}{d\tau}\right) \approx (1, v/c)$$

$$G^{\alpha\beta}(\{g^{\mu\nu}\}) = -\frac{8\pi G}{c^2} T^{\mu\nu}$$

nonlinear in $g^{\mu\nu}$

THE EVOLUTION OF OUR UNIVERSE

COSMOLOGY — WHY GR??

- universe is compact object

$$R \simeq \frac{2GM}{c^2} = \frac{2G}{c^2}\,\frac{4\pi}{3}\,\rho R^3$$

schwarzschild radius

$$\left\{ \begin{array}{l} v = H_0 d \\ c = H_0 R \rightarrow R = c/H_0 \end{array} \right.$$

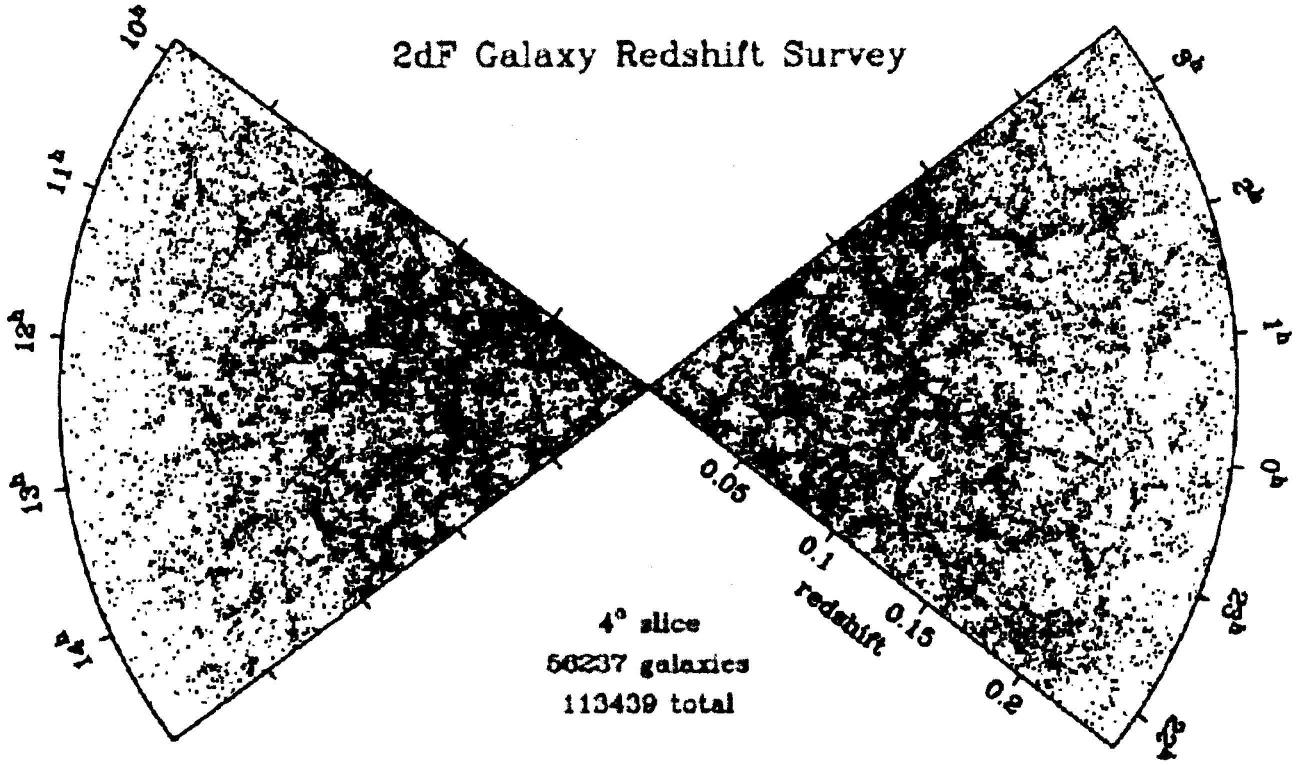
$$\rho \simeq \frac{3H_0^2}{8\pi G} \stackrel{D}{=} \rho_c \qquad \text{critical density}$$

- $H_0 = 100\, h \quad km\, s^{-1}\, Mpc^{-1}$

  $h = 0.71 \pm 0.04 \qquad (WMAP)$

  $H_0 = (2.3 \pm 0.1) \times 10^{-18}\ s^{-1}$

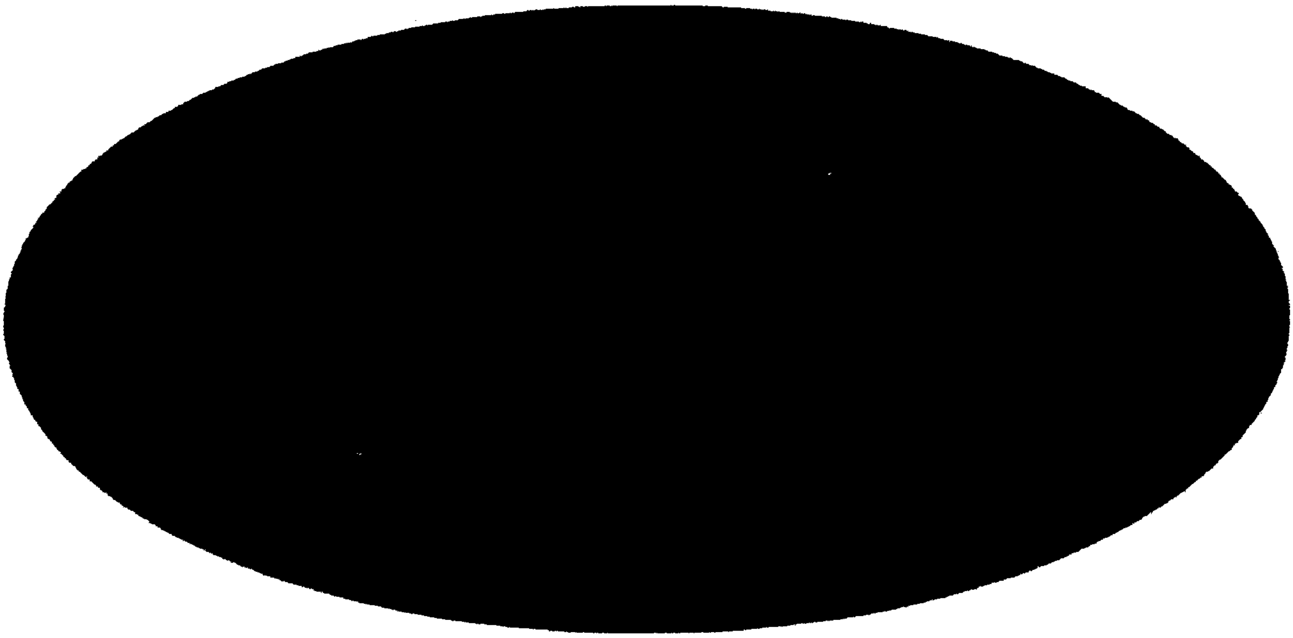
- $\rho_c \simeq 10^{-29}\quad g\, cm^{-3}$



TYPE	$\Omega = \rho/\rho_c$	
MATTER (Ω_m)	0.27	
luminous baryons	0.006	$\Omega_b \simeq 0.04$
dark "	0.038	
WIMPS	0.23	unknown
DARK ENERGY (Ω_Λ)	0.73	unknown
TOTAL $\Omega_M + \Omega_\Lambda$	1.02 ± 0.02	flat geometry

Matter distribution isotropic also within redshift classes

WMAP IMAGE CMB, $\lambda = 3.2$ mm.



- $T = 2.725\,K$
- monopole & dipole subtracted, foreground emissions not yet

black: $-200\,\mu K$ red: $+200\,\mu K$

Energy densities

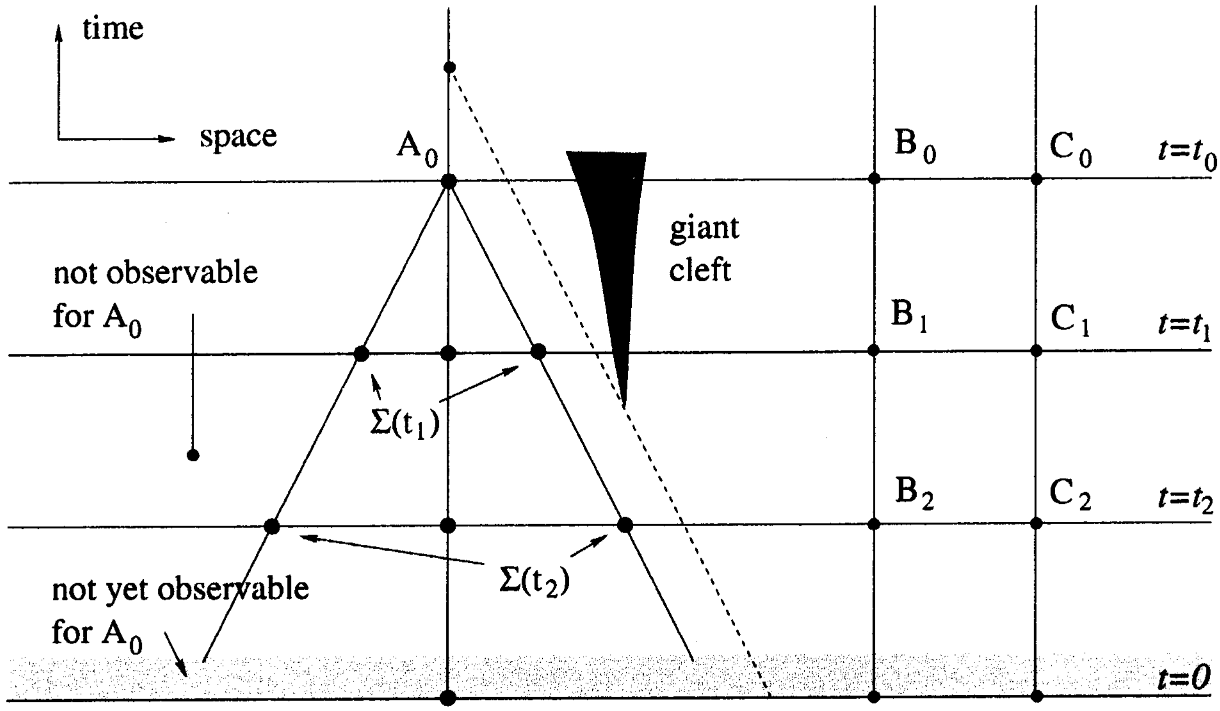
$$\varepsilon_{matter} = \Omega_m \rho_c c^2 = \underline{\underline{2.4 \times 10^{-9} \text{ erg cm}^{-3}}}$$

$$\varepsilon_{CMB} = \frac{4\sigma}{c} \underset{\uparrow\, 2.725K}{T^4} = 4.2 \times 10^{-13} \text{ erg cm}^{-3}$$

$$\varepsilon_{\nu\bar{\nu}} = 2.8 \times 10^{-13} \quad ''$$

$$\varepsilon_{radiation} = \underline{\underline{7 \times 10^{-13} \text{ erg cm}^{-3}}}$$

THE SPACETIME OF OUR UNIVERSE



- past lightcone : "nested shells"
- Isotropy → shells Σ are homogeneous ($\Sigma_1 \neq \Sigma_2$
- Cosmological principle → spaces t = constant are homogeneous
- Rest $=_D$ not moving w.r.t. Hubble flow

∴ spatial coordinates galaxies are constant (we ignore their small peculiar velocities)

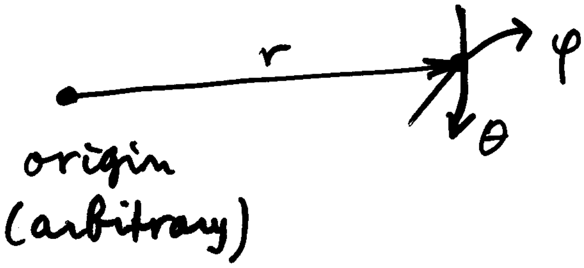
∴ worldlines vertical

- Co-ordinate distance B & C is constant
  geometrical distance B & C grows (Expansion!)
- cosmological principle needed, seems OK, but may prove incorrect in the future!

- <u>SYMMETRY ARGUMENTS</u> $\rightarrow$ METRIC

$$ds^2 = (dx^0)^2 - S^2(t)\left[dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right]$$

r, ϑ, φ spatial coordinates



$$ds^2 = c^2 dt^2 - \underbrace{S^2 dl^2}$$

$\left(\begin{array}{c}\text{Physical distance}\\ \text{in space}\end{array}\right)^2$

— actually $k = 0, \pm 1$ types, but $\Omega_m + \Omega_\Lambda = 1 \rightarrow$ flat
(without proof)

- <u>EVOLUTION EQUATIONS</u>

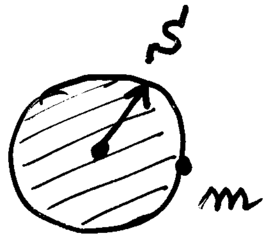
1. $(\rho c^2 S^3)^{\cdot} + p\,(S^3)^{\cdot} = 0$ $\qquad$ $dU + p\,dV = 0$
expansion is adiabatic.

2. $$\left(\frac{\dot S}{S}\right)^2 = \underbrace{\frac{8\pi G\rho}{3}}_{\text{classical gravity}} + \underbrace{\frac{\Lambda c^2}{3}}_{\text{cosmological constant}} \rightarrow S \propto \exp(t)$$

- <u>Three unknowns</u>: ρ, p, S

need e.g. $p(\rho)$, or rather $\rho(S)$

CLASSICAL GRAVITY



NEWTON:

$$m \ddot{s} = -G m \left(\frac{4\pi}{3} s^3 \rho \right) \cdot \frac{1}{s^2}$$

$$\ddot{s} = -\frac{4\pi G}{3} \rho s$$

$\rho s^3 = \rho_0 s_0^3$

$$\ddot{s} = -\frac{4\pi G \rho_0 s_0^3}{3} \frac{1}{s^2}$$

$* \dot{s}$ & integrate

$$\dot{s}^2 = \frac{8\pi G \rho_0 s_0^3}{3} \cdot \frac{1}{s} + \text{const}$$

$= 0$

$\rho_0 s_0^3 / s = \rho s^2$

COSMOLOGICAL CONSTANT

- $T^{\mu\nu}$ of classical fluid in rest-frame:

$$T^{\mu\nu} = \frac{1}{c^2}\begin{pmatrix} \rho c^2 & & & \phi \\ & p & & \\ & & p & \\ \phi & & & p \end{pmatrix}$$

- Nb pressure p is form of energy and if $p \sim \rho c^2$ it generates gravity

  This causes collapse $NS \to BH$!

- Nb $\frac{dp}{dr}$ supports star ; $p \to$ gravity

- Accept this $T^{\mu\nu}$ as $T^{\mu\nu}$ of vacuum

  $\rho = \rho_v \quad , \quad p = p_v$

- Vacuum identical in all inertial frames

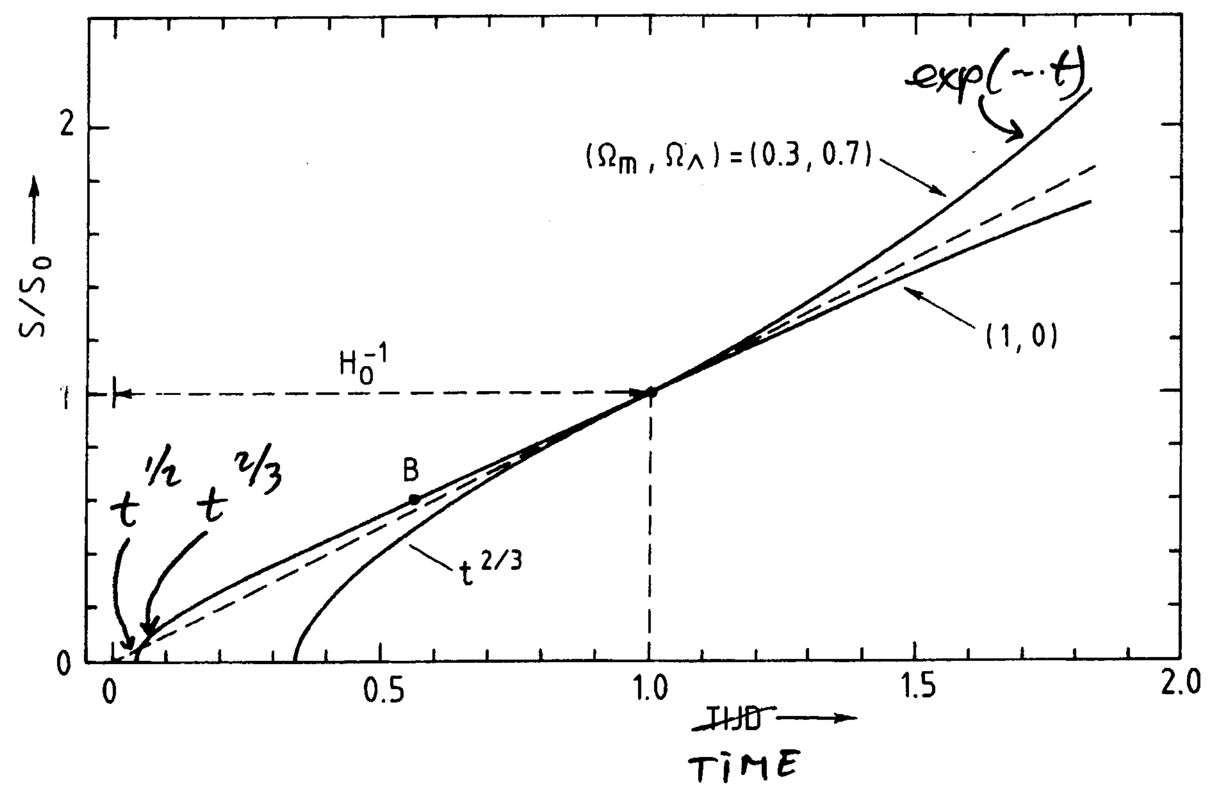
$$T^{\mu\nu} \propto \eta^{\mu\nu} = \begin{pmatrix} 1 & & & \phi \\ & -1 & & \\ & & -1 & \\ \phi & & & -1 \end{pmatrix}$$

$$\therefore \; T^{\mu\nu} = \rho_v \begin{pmatrix} 1 & & & \phi \\ & -1 & & \\ & & -1 & \\ \phi & & & -1 \end{pmatrix} \quad \text{and} \quad p_v = -\rho_v c^2 < 0!$$

- ρ_v : universal attraction : space tries to contract

  $p_v < 0$: space blows itself up

- Ultimate explanation : Quantum gravity



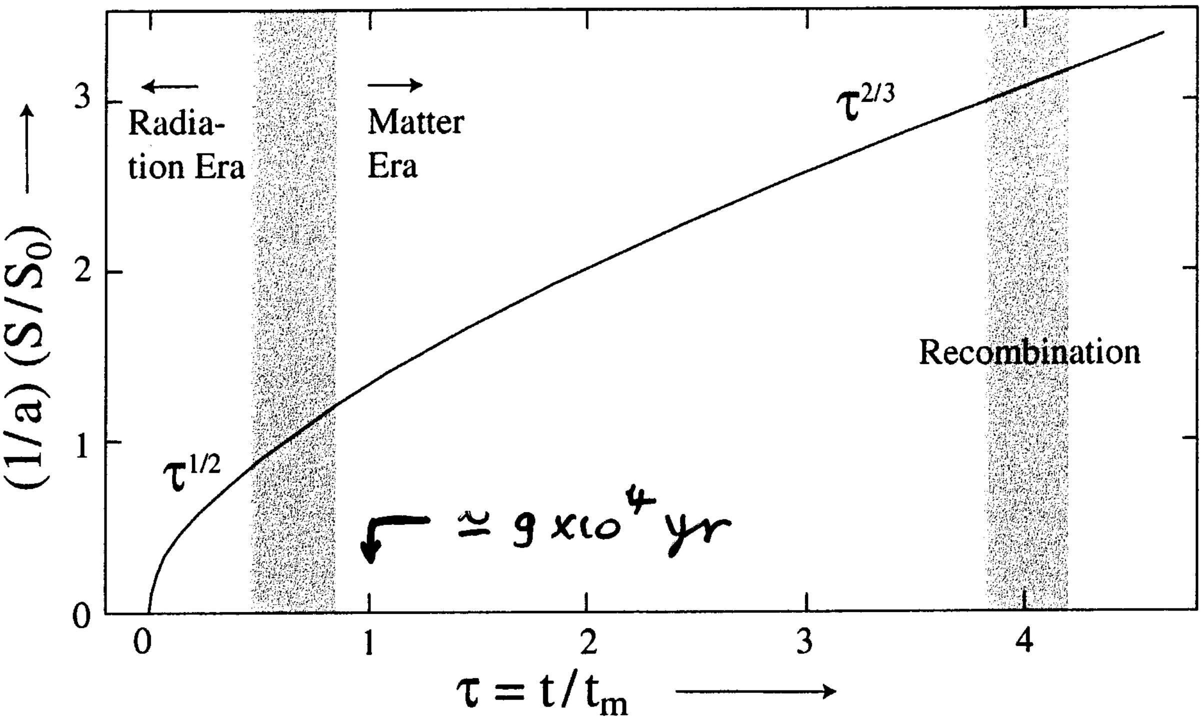
$$\rho S^3 = \text{constant} \Rightarrow$$

$$\dot{u} = H_0 \left(\Omega_m u^{-1} + \Omega_\Lambda u^2 \right)^{1/2} \qquad u = S/S_0$$

$$\left[\begin{array}{l} \Omega_m = \rho_0 / \rho_c \simeq 0.27 \\ \Omega_\Lambda = \rho_v / \rho_c \simeq 0.73 \end{array} \right]$$

- $u \ll 1 \rightarrow \dot{u} \propto u^{-1/2} \rightarrow u \,(:)\, t^{2/3}$
- $u \gg 1 \rightarrow \dot{u} \propto u \rightarrow u \,(:)\, \exp(--t)$
- singularity $u = 0$ must occur if $\Omega_\Lambda < 1$
- Age $\simeq H_0^{-1} \simeq 14$ Gyr.

$1/a \simeq 3300$



$T_{rad} = T_{mat}\ (:)\ S^{-1}$

$\varepsilon_{rad} > \varepsilon_{mat}$

$\varepsilon_{rad} \propto S^{-4} \propto T^4$

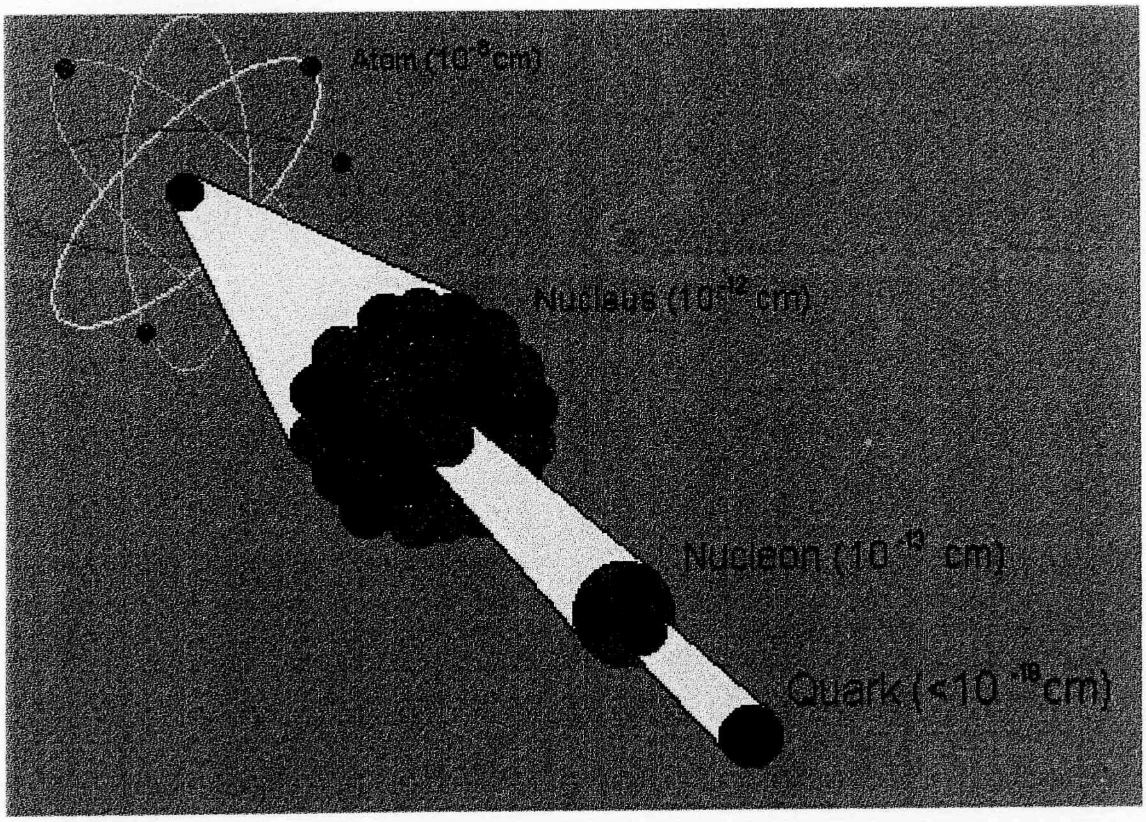
$T_{rad} \simeq 3000\,K$

$S_0/S \simeq 1100$

<u>Universe very homogeneous</u>

<u>No galaxies</u>

THE BIG BANG



Elementary Particles

Quarks 6 types, 3 colour charges, fractional electric charge + antiquarks $\longrightarrow$ 36 in total

Leptons e^- ν_e, μ^- ν_μ, τ^- ν_τ

+ antiparticles 12 in total

Gauge bosons gluons
vector bosons
photon
graviton

---

nonbaryonic matter ? supersymmetric particles, WIMPS (-no EM), ----

Table 2. Overview of the evolution of the universe

age (s)	temperature (K)	size (S/S_0)	composition[a]: baryons	leptons	gauge bosons	
$< 10^{-7}$	$> 10^{13}$	$< 2 \times 10^{-13}$	$\mathbf{q\,\bar{q}}$	$\boldsymbol{\ell\,\bar{\ell}}$	$\boldsymbol{\gamma, g, W^{\pm}, Z^{0}}$, ..	
10^{-6}	5×10^{12}	5×10^{-13}	$\mathbf{p\,\bar{p}, n\,\bar{n}}$, ..	$\boldsymbol{\ell\,\bar{\ell}}$	$\boldsymbol{\gamma, g}$	
10^{-4}	10^{12}	3×10^{-12}	p, n	$\mathbf{e^{-}e^{+}}, \boldsymbol{\nu\,\bar{\nu}}$	$\boldsymbol{\gamma, g}$	*
10^{2}	10^{9}	3×10^{-9}	p, n	$e^{-}, \boldsymbol{\nu\,\bar{\nu}}$	$\boldsymbol{\gamma, g}$	*
10^{3}	3×10^{8}	10^{-8}	^{1}H, ^{4}He	$e^{-}, \boldsymbol{\nu\,\bar{\nu}}$	$\boldsymbol{\gamma, g}$	
$> 10^{13}$	< 3000	$> 10^{-3}$	H, He atoms	$\boldsymbol{\nu\,\bar{\nu}}$	$\boldsymbol{\gamma, g}$	
			$\Downarrow$	$\Downarrow$	$\Downarrow$	
4×10^{17}	3	1	galaxies	neutrino, microwave and graviton background		

[a] Boldface printed particles have comparable number densities, and these are about 10^9 times larger than those of the other particles on the same line.

<u>REACTIONS</u> $A + B \longleftrightarrow C + D$

$\hookrightarrow P + Q$ -----

whole network

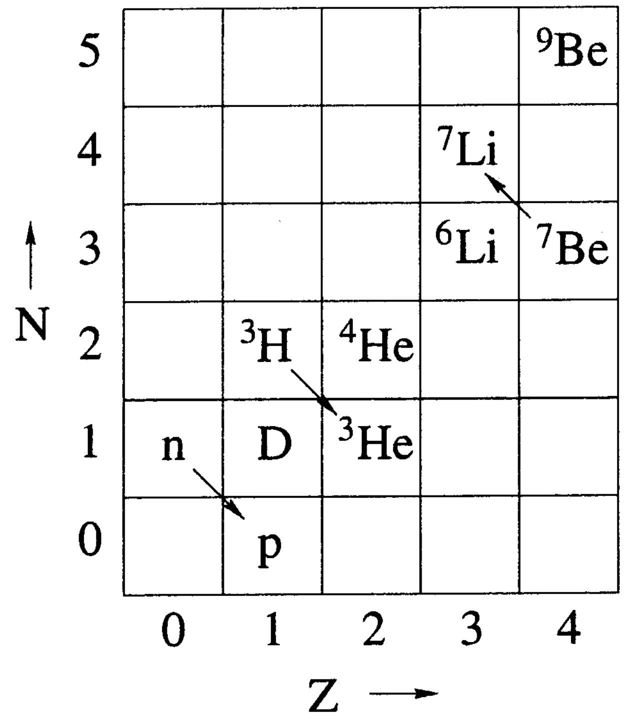
- Available time : $\sim (\dot{S}/S)^{-1} \sim$ age universe

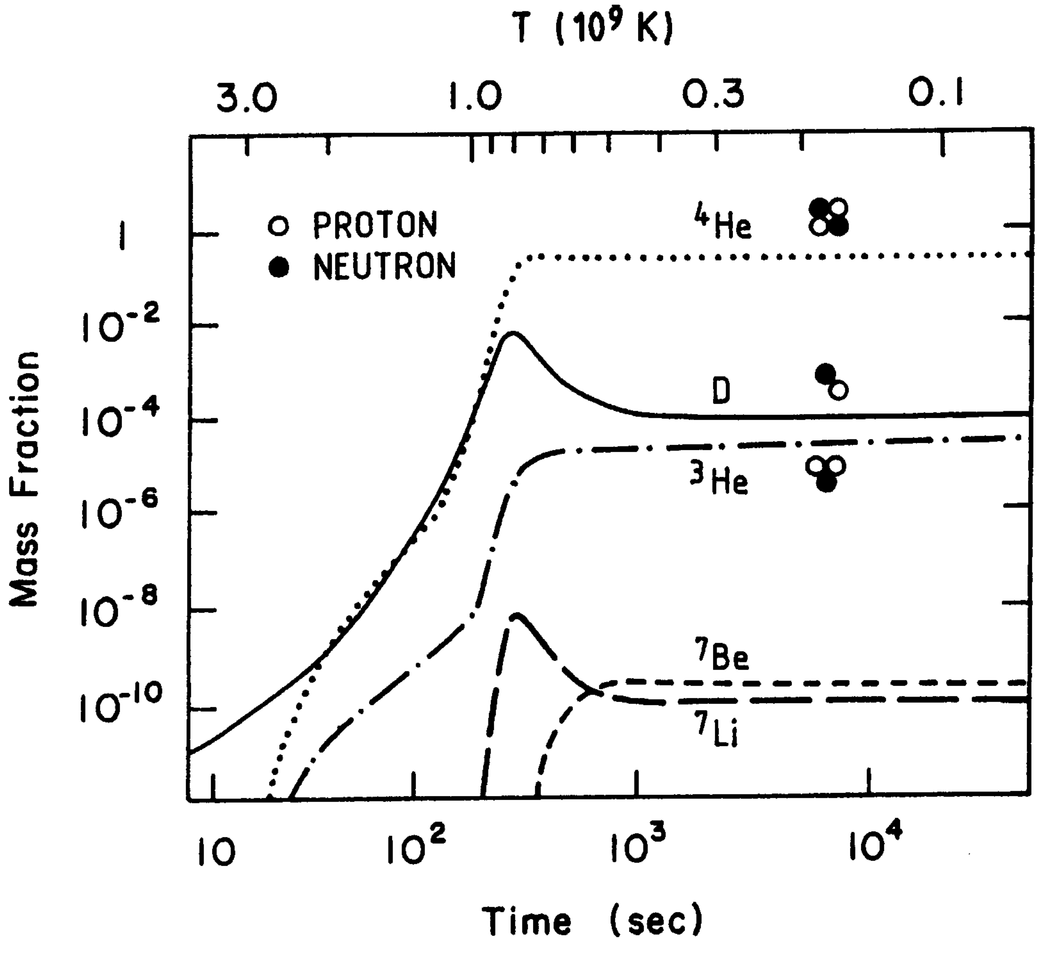
- thermal equilibrium $\longrightarrow$ freeze out

* $n\bar{n}$ asymmetry $\sim 10^{-8}$ unknown origin

* $\frac{n}{p} = \exp(\Delta E/kT) = 1 \longrightarrow$ freeze out at $\frac{n}{n+p} = 0.16$

decrease (n-decay) to ~ 0.13 at onset synthesis elements.



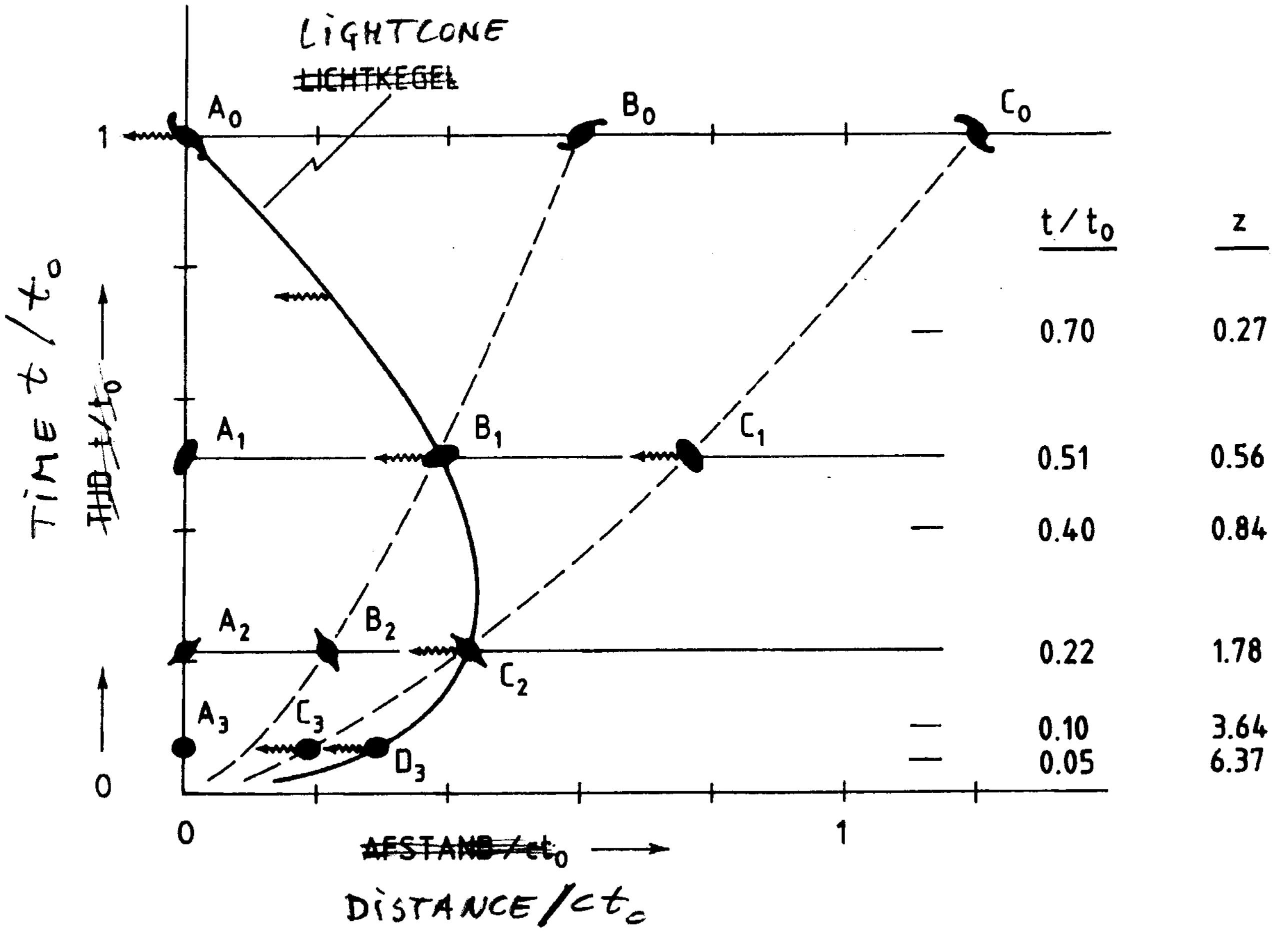


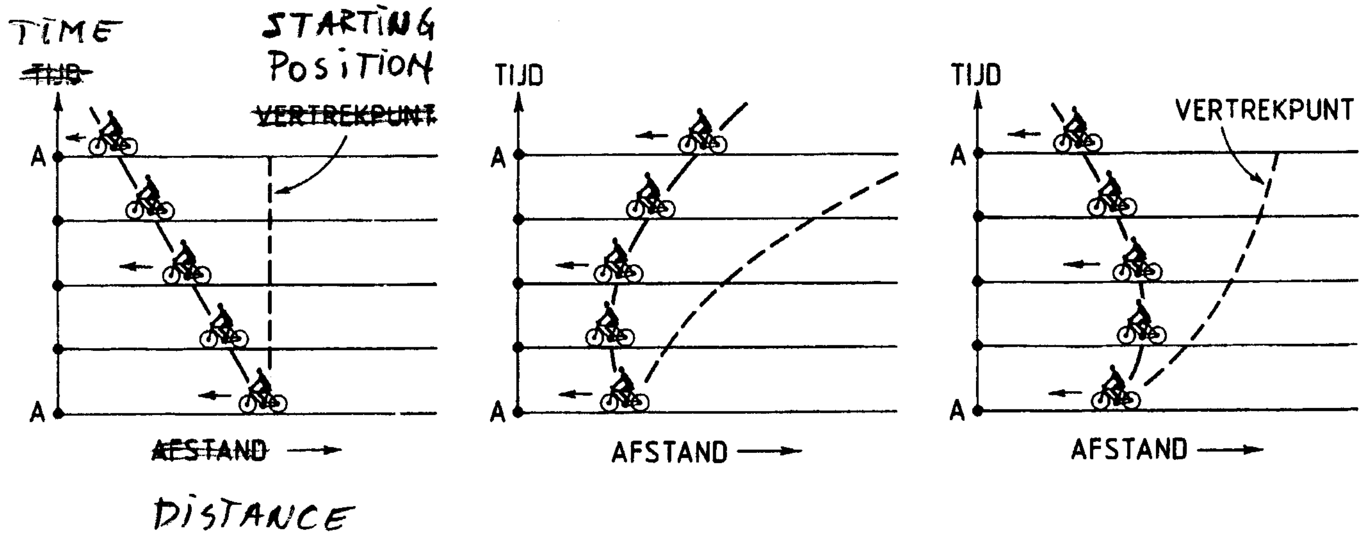
– All n's end up in ^4He → $Y = 2 \times 0.13 = 0.26$

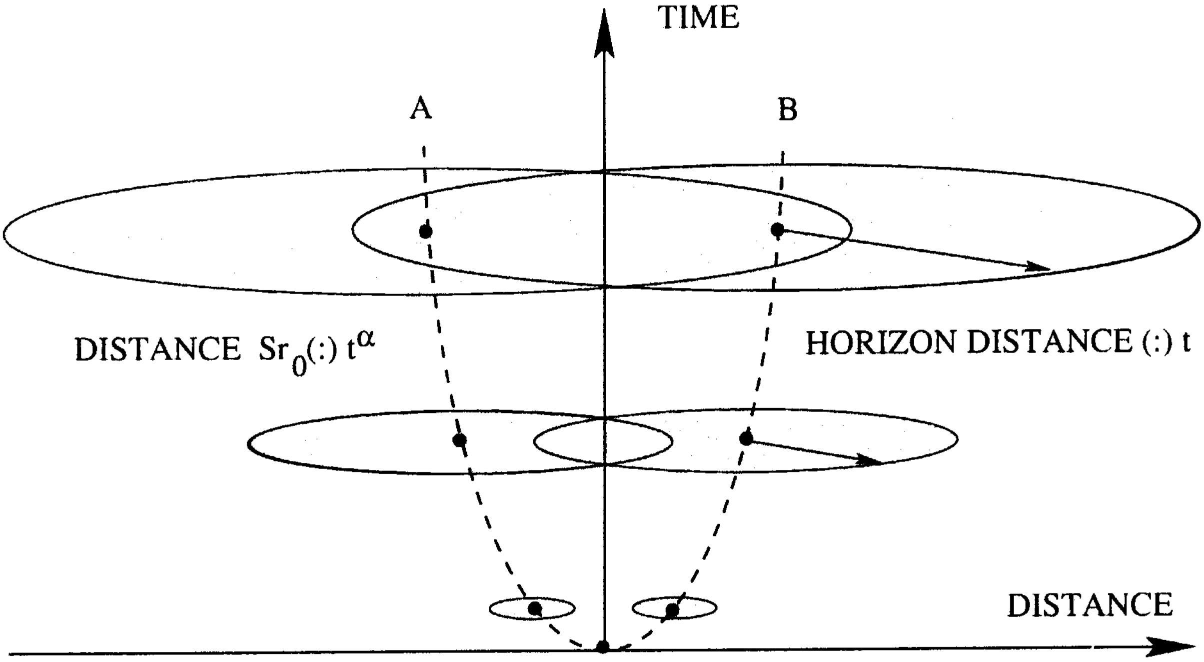
– no time for synthesis heavier elements !

OBSERVATIONAL ISSUES

1. how do we observe the universe
2. the horizon (problem)
3. a common misconception
4. the angular correlation spectrum of the C.M.B.


LIGHTCONE
~~LICHTKEGEL~~
A_0
B_0
C_0
1
0
TIME t/t_0
~~TIJD t/t_0~~
A_1
B_1
C_1
A_2
B_2
C_2
A_3
C_3
D_3
0
1
~~AFSTAND / ct_0~~
DISTANCE / ct_0
t/t_0
z
0.70 0.27
0.51 0.56
0.40 0.84
0.22 1.78
0.10 3.64
0.05 6.37


TIME
~~TIJD~~
STARTING POSITION
~~VERTREKPUNT~~
A
A
~~AFSTAND~~
DISTANCE
TIJD
A
A
AFSTAND
TIJD
VERTREKPUNT
A
A
AFSTAND



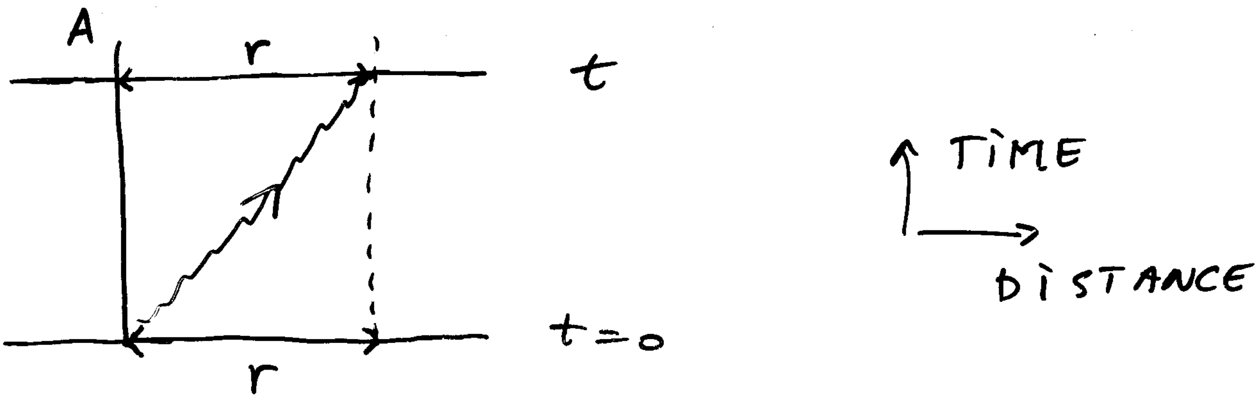
- HORIZON delimits sphere of influence around each observer

  light emitted by matter outside horizon has not (yet) been able to reach the observer

- VISIBLE UNIVERSE = sphere inside horizon of the observer

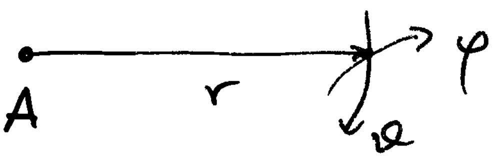
- HORIZON PROBLEM only solved with advent of inflation theory

DISTANCE TO HORIZON



$$ds^2 = c^2 dt^2 - S^2\left[dr^2 + \cancel{r^2(\ldots)}\right]$$

$$d\theta = d\varphi = 0$$

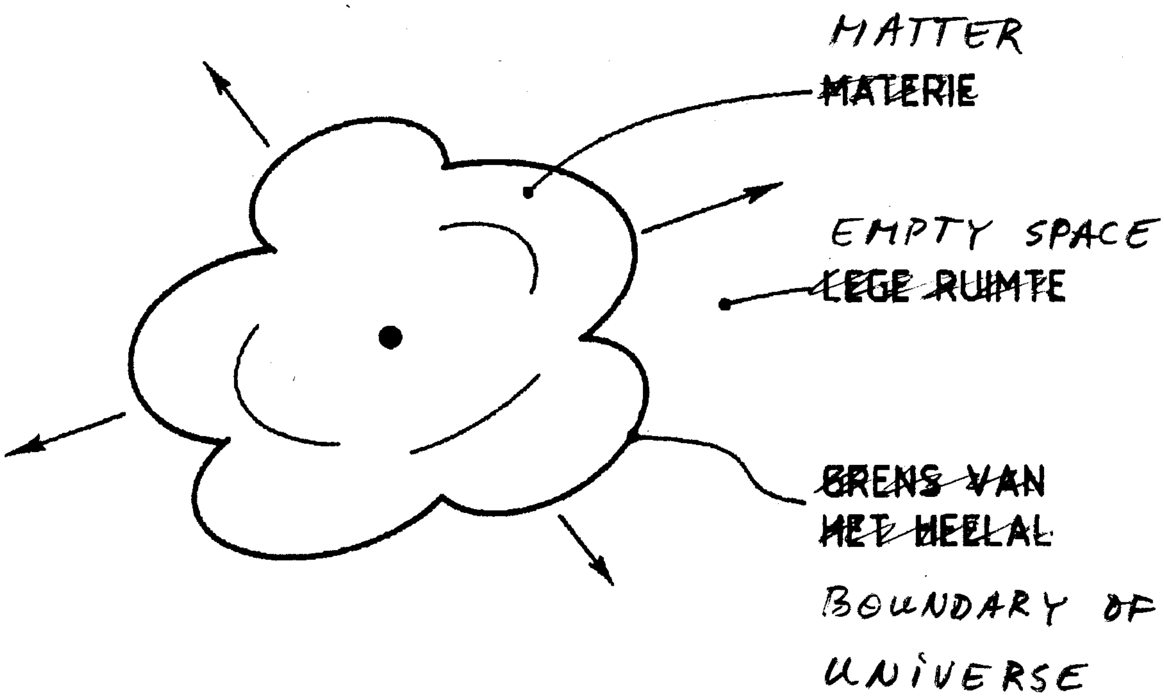


$ds = 0$ (photon) $\longrightarrow$ $dr = c\,dt/S$

$$\therefore \quad r = c\int_0^t \frac{d\tau}{S} \quad \longrightarrow \text{distance} = Sr = cS\int_0^t \frac{d\tau}{S}$$

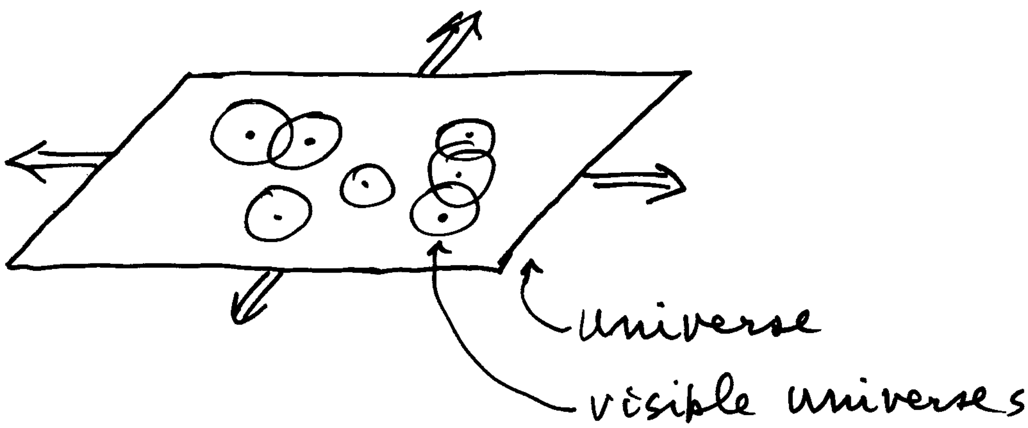
- take $S \propto t^\alpha$
- distance $= \dfrac{ct}{1-\alpha} = \begin{cases} 3ct & \alpha = 2/3 \\ 2ct & \alpha = 1/2 \end{cases}$
- photons superluminal ??

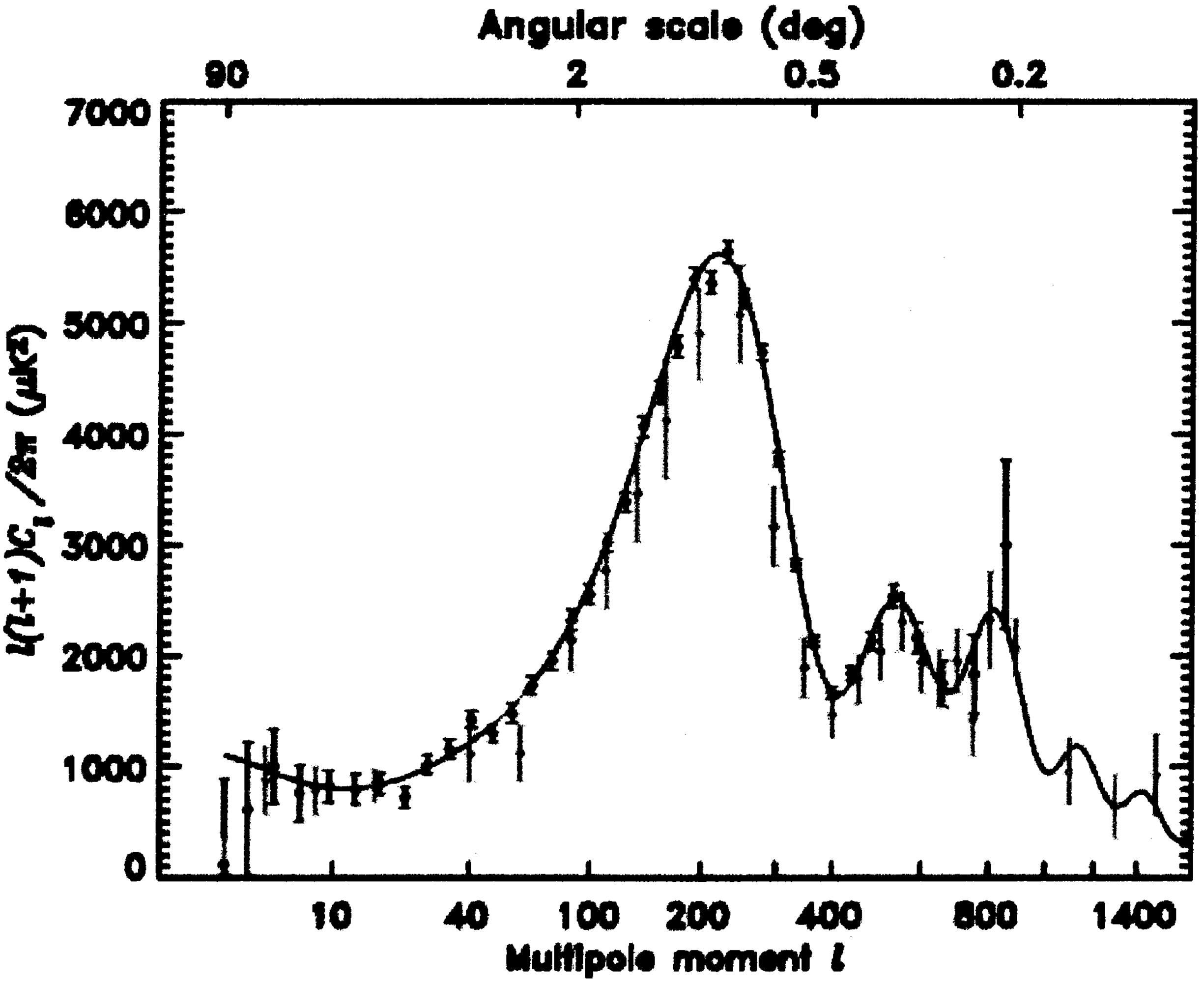
A COMMON MISCONCEPTION: THE BIG BANG AS A POINT EXPLOSION



- $\bar{\rho}$ + AGE $\Rightarrow$ τ to boundary of explosion is $\ll 1$

- universe isotropic $\Rightarrow$ we are in the center of a spherically symmetric explosion





POWER SPECTRUM CMB FROM WMAP DATA

$\mathbf{I}$ = PREVIOUS RESULTS

① $\Delta T = T(\vartheta, \varphi) - \langle T \rangle$

$\langle \cdot \rangle = \frac{1}{4\pi} \int d\Omega$

② $C_\ell = 2\pi \int_0^\pi \langle \Delta T(\underline{u}) \Delta T(\underline{v}) \rangle \big|_{\underline{u}\cdot\underline{v} = \cos\vartheta} P_\ell(\cos\vartheta) \sin\vartheta \, d\vartheta$

③ Figure shows "power spectrum" $\ell(\ell+1)C_\ell / 2\pi$

THE PLAYERS

(1). SCALAR FIELD φ

At end of inflation φ & $\delta\varphi \longrightarrow \rho$ & $\delta\rho$
(matter)

(2). "$be\gamma$ FLUID"

- p, ^4He, e^-, γ
- $\Omega_b = 0.04$
- tightly coupled system until t_{rec}
- high 'sound' speed $c/\sqrt{3}$ ($10^9\,\gamma$ per baryon)

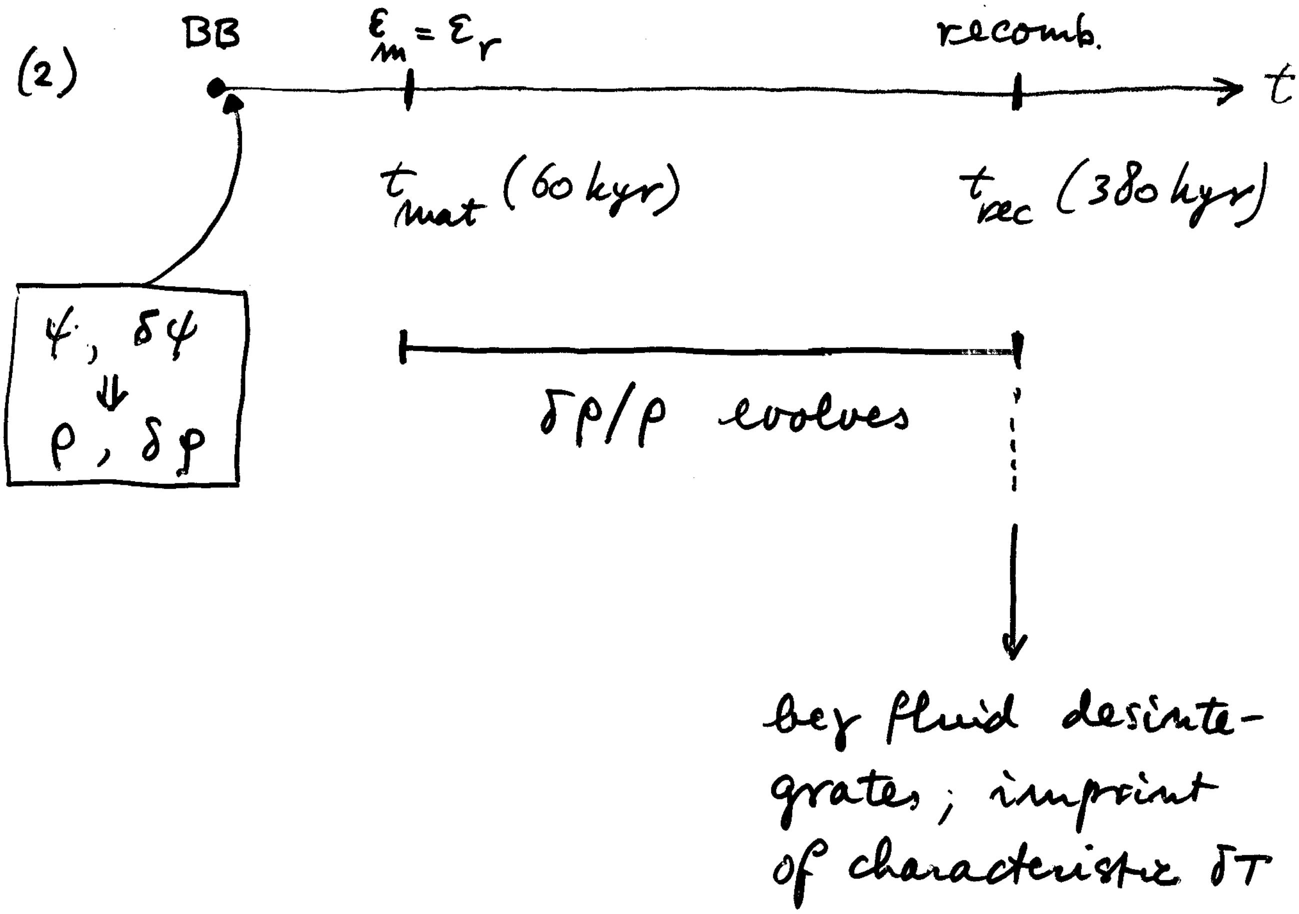
(3). NONBARYONIC DARK MATTER "DM"

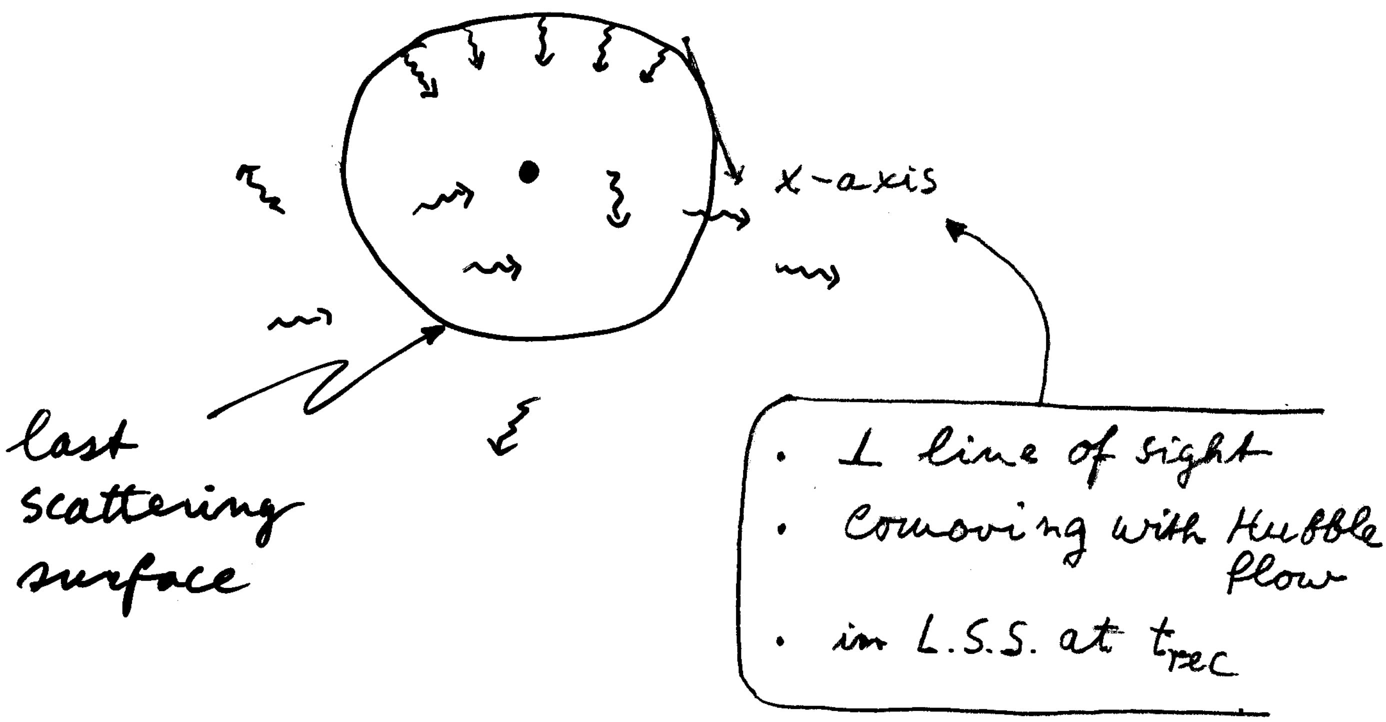
- no EM interactions
- $\Omega_{dark} = 0.23 \longrightarrow$ DM fixes ϕ, $\delta\phi$
- Cold $\longrightarrow$ low 'sound' speed ("CDM model")

THE PLOT (1)

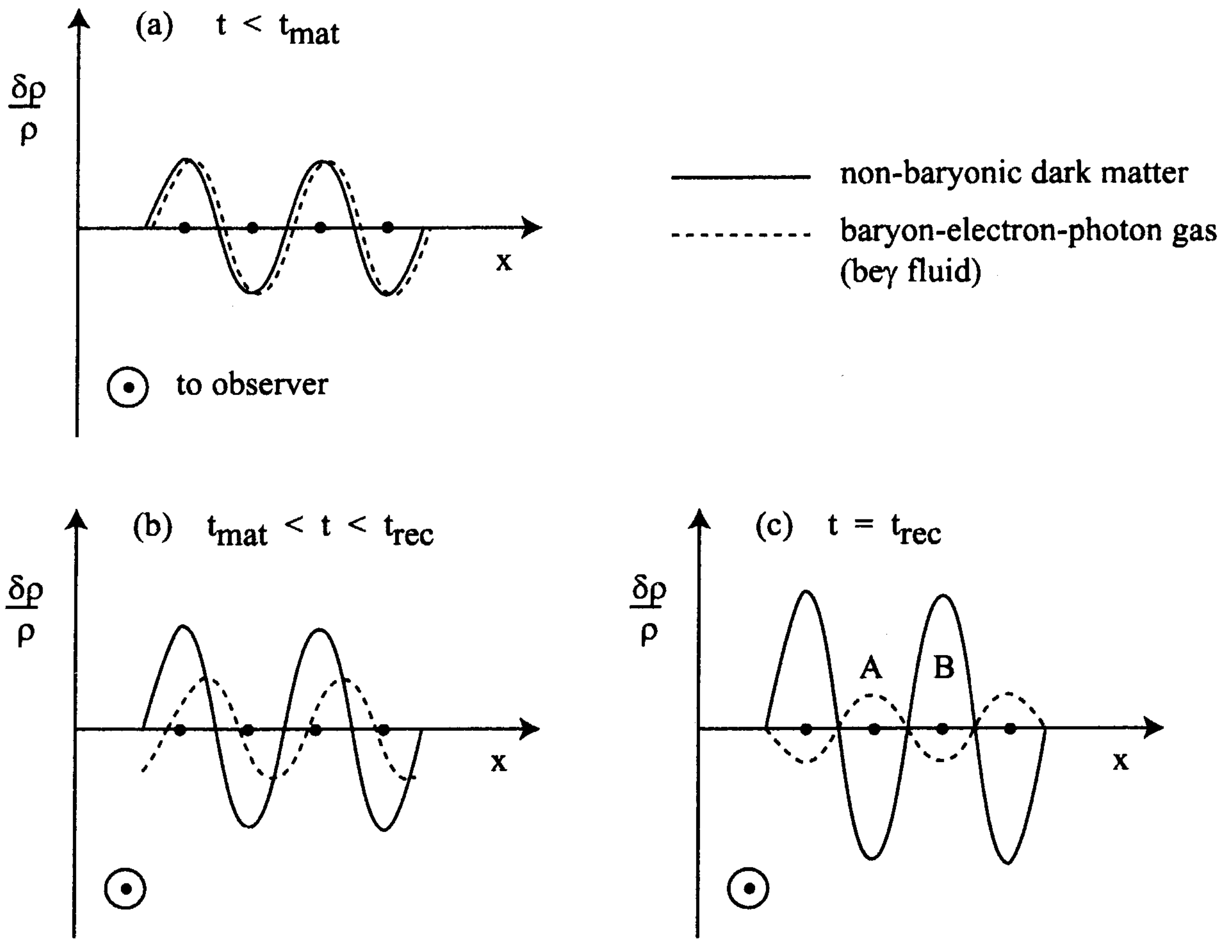
(1) DM and bry communicate only through gravity

(2)





THE PLOT (2)



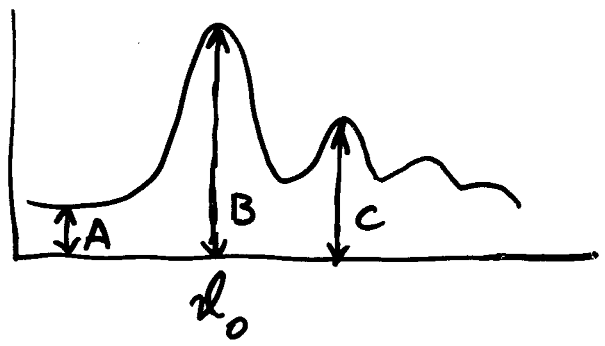
- $\delta\rho/\rho = \Sigma$ fourier modes (only in L.S.S.)
- $(\delta\rho/\rho)_{DM}$ grows, but modes do not move
- $(\delta\rho/\rho)_{be\gamma}$ damps, but modes do move
- All beγ modes travel same distance D; shown is that mode for which $D = \lambda/2$ at t_{rec}
- $$\frac{\delta T}{T} = \begin{cases} \delta\phi/c^2 & >0 \text{ in A} \quad <0 \text{ in B} \\ \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_{be\gamma} & >0 \text{ in A} \quad <0 \text{ in B} \end{cases}$$

distance of L.S.S.

$$\theta = \frac{\lambda/2}{d} = \frac{D}{d} \rightarrow \frac{D}{(2n+1)d} = \frac{0.6^\circ}{2n+1}$$

ANALYSIS WMAP DATA

- complex modelling required



WMAP + HST Key H_0 + SNIa $\longrightarrow$ $\Omega_M + \Omega_\Lambda = 1$

B/C $\longrightarrow$ $\Omega_b h^2$

B/A $\longrightarrow$ $\Omega_M h^2$

ℓ_0 $\longrightarrow$ h

INFLATION

- Most important <u>theoretical</u> development in cosmology of last 25 yr.

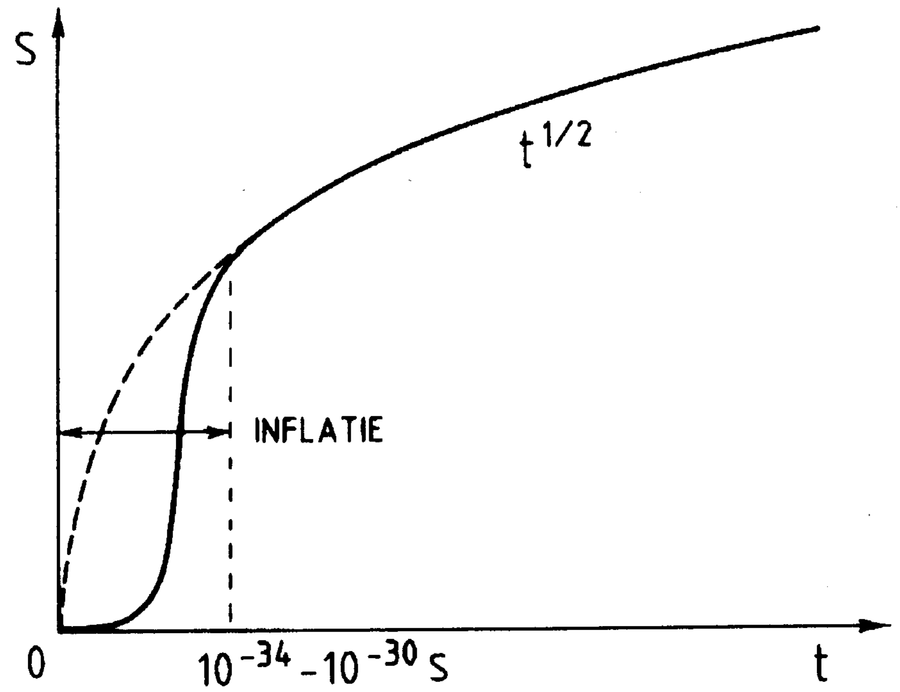
- Successes of F.R.W. Universe
  - expansion velocities distant galaxies
  - the relics: CMB + chemical composition (H, D, ^4He, ^3He, ^7Li)

- BUT
  - why is universe spatially flat?
  - Horizon problem?
  - why expansion?

  INFLATION

  [• $m\,\bar{m}$ asymmetry
  • vacuum energy]

THE ESSENCE OF INFLATION



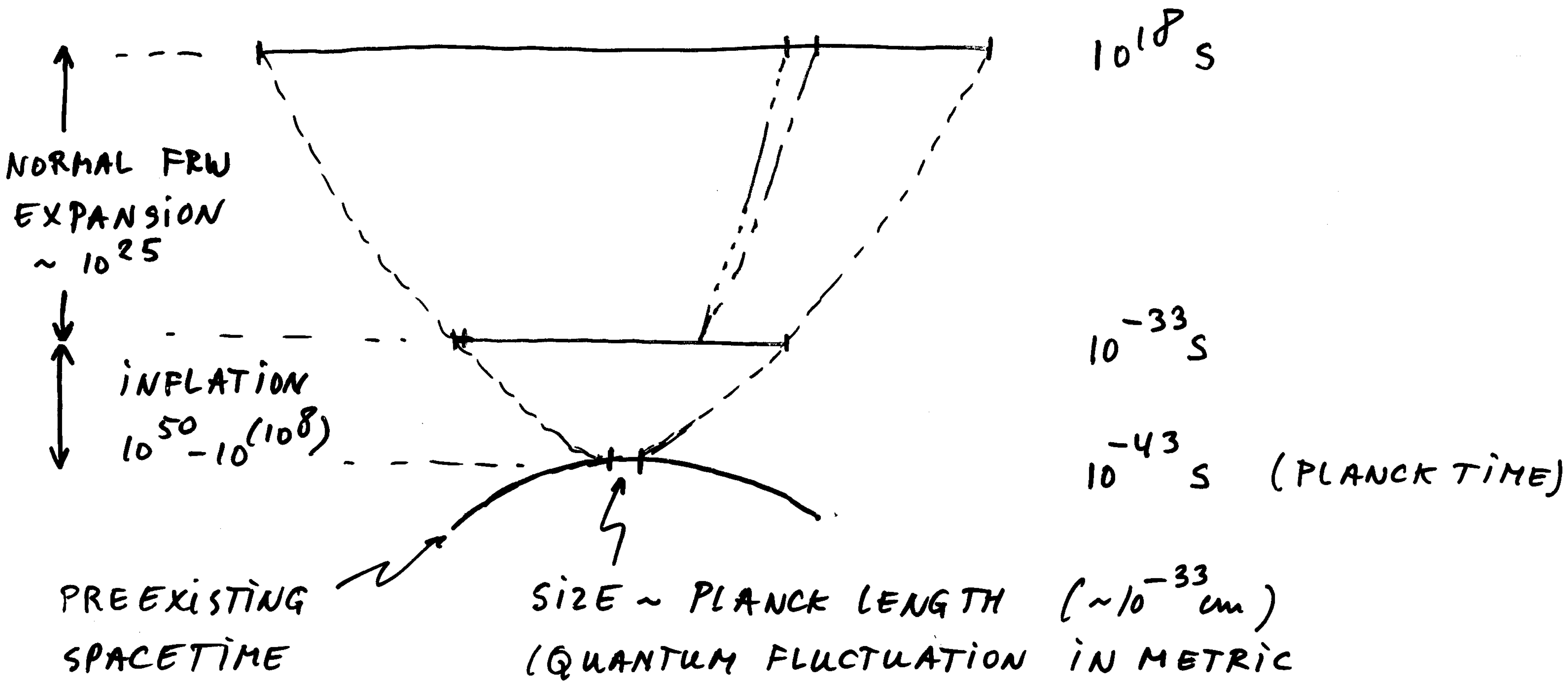
① Distance to horizon $= c\,S \int_0^t \frac{d\tau}{S(\tau)} \quad \propto t$
for $S (:) t^{\alpha}$

change $S(t)$ near $t=0$ $\Rightarrow$ much larger horizon distance!
($S(t)$ at later times virtually fixed)

② $S\,(:)\,t^{\alpha} \longrightarrow \dot{S} \uparrow \infty$ as $t \downarrow 0$

expansion speed infinite $\longrightarrow$ universe desintegrates into separate parts

③ Take universe initially extremely small
wait size/c sec $\rightarrow$ causal contact
blow up to huge proportion $\rightarrow$ horizon distance also huge


NORMAL FRW EXPANSION
~ 10^25
INFLATION
10^50 - 10^(10^8)
10^18 S
10^-33 S
10^-43 S (PLANCK TIME)
PREEXISTING SPACETIME
SIZE ~ PLANCK LENGTH (~10^-33 cm)
(QUANTUM FLUCTUATION IN METRIC

INFLATION / TECHNICAL DETAILS

- Guth (1981): scalar fields may cause inflation
- scalar fields correspond to hypothetical, heavy, zero-spin bosons that may occur in GUTs.
- QM in expanding spacetime

$$\left.\begin{aligned} E^2 &= (\underline{p}c)^2 + (mc^2)^2 \\ E &\to i\hbar \frac{\partial}{\partial t} \\ \underline{p} &\to -i\hbar \underline{\nabla} \end{aligned}\right\} \quad \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \mu^2\right)\psi = 0$$

$$\mu = mc/\hbar$$

Klein-Gordon equation for wave function free relativistic $s=0$ boson

$$\eta^{\alpha\beta}\psi_{,\alpha,\beta} + \mu^2\psi = 0 \quad \rightarrow \quad g^{\alpha\beta}\psi_{:\alpha:\beta} + \mu^2\psi = 0$$

ordinary covariant derivative

- assume $\underline{\nabla}\psi = 0$

$$\boxed{\psi_{,0,0} + \frac{3}{c}\frac{\dot{S}}{S}\psi_{,0} + \mu^2\psi = 0}$$

Damped harmonic oscillator

$_{,0} = \partial/\partial t$

- Evolution universe with scalar field

$$\left(\frac{\dot{S}}{S}\right)^2 = \frac{8\pi G \rho}{3} + \frac{\Lambda c^2}{3}$$

unimportant in early univ.

replace by energy density ψ field

add curvature term kc^2/S^2

[$k=0$ flat, $k=+1$ spherical univ.
$k=-1$ hyperbolic univ.]

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{kc^2}{S^2} = \frac{4\pi G}{3}\left\{\underbrace{\psi_{,0}^2 + \mu^2\psi^2}_{} + \cancel{|\nabla\psi|^2}\right\}$$

think of energy $\dot{\phi}^2 + \omega^2\phi^2$
of harmonic oscillator!

- Make dimensionless

$M = (\hbar c/G)^{1/2}$ = Planck mass (2.2×10^{-5} g)
($r_{schw.} = \lambda_{compt}$)

$L_p = \hbar/Mc = 1.6 \times 10^{-33}$ cm

$t_p = L_p/c = 5.4 \times 10^{-44}$ s

substitute $G = \frac{\hbar c}{M^2}$, then set $\hbar = c = 1$

$$\boxed{\begin{aligned} &\ddot{\psi} + 3H\dot{\psi} + m^2\psi = 0 \\ &H^2 + \frac{k}{S^2} = \frac{4\pi}{3M^2}\left(\dot{\psi}^2 + m^2\psi^2\right) \end{aligned}}\qquad \begin{aligned} &1 \\ &2 \end{aligned}$$

p.m. $\nabla\psi = 0$ and $H = \dot{S}/S$

closed set equations for ψ and S

- Initial conditions $\Rightarrow$

- Solution of equations

Assume $H \gg m$ (strong damping) $\Big\} \Rightarrow \ddot{\psi} \simeq 0$; $\dot{\psi} \ll m\psi$

$H = \dot{S}/S \sim$ constant $\Rightarrow$ $S(t)$ exponential and k/S^2 rapidly ignorable

$$\boxed{\begin{aligned} &3H\dot{\psi} + m^2\psi = 0 \\ &H^2 = \frac{4\pi m^2}{3M^2}\psi^2 \end{aligned}}\qquad \begin{aligned} &1 \\ &2 \end{aligned}$$

- 1 $* \psi \longrightarrow m^2\psi^2 = -3H\psi\dot{\psi} \longrightarrow$ in 2

$$H^2 = \frac{4\pi}{3M^2}\left(-3H\psi\dot{\psi}\right) \longrightarrow H = -\frac{4\pi}{M^2}\psi\dot{\psi} \quad 3$$

$$\therefore \quad \frac{\dot{S}}{S} + \frac{4\pi}{M^2}\psi\dot{\psi} = 0 \longrightarrow \text{integrate!}$$

– Initial conditions at $t = t_P$

- All energy "quantum bubble" $\sim L_P$ may contain resides in ~~one~~ scalar field φ

[Nb: Usually energy divided over many different fields, but these regions do not inflate]

$$Mc^2 \cdot t_P \sim \hbar \qquad \text{(Heisenberg)}$$

$$\therefore \quad \underbrace{\cancel{\dot{\varphi}^2} + m^2\varphi^2}_{\text{energy density}} \sim \frac{Mc^2}{L_P^3} = M^4$$

strong damping $\rightarrow \dot{\varphi}$ very small

$$\therefore \quad \boxed{\varphi_P \equiv \varphi(t_P) = M^2/m} \qquad \dot{\varphi}_P \text{ not needed}$$

further: $\boxed{S_P = S(t_P) = L_P}$

– Why $\nabla\varphi \simeq 0$?

$$|\nabla\varphi|^2 \leq M^4 \rightarrow \delta\varphi \simeq |\nabla\varphi| L_P < M^2 M^{-1} = M$$

$$\therefore \delta\varphi/\varphi \sim M/(M^2/m) = m/M \ll 1$$

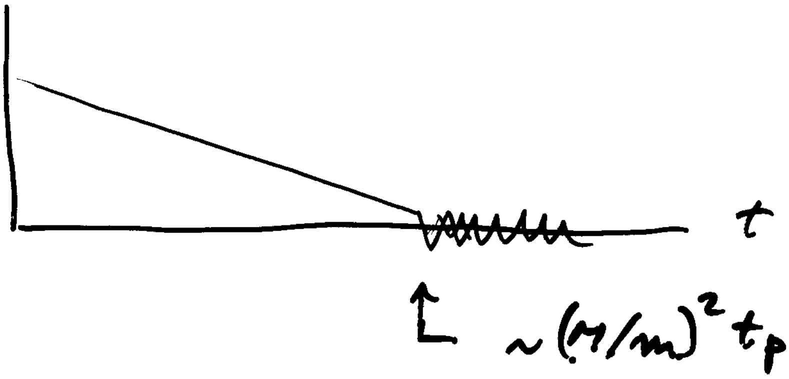
scalar boson mass $\ll$ Planck mass

$\therefore$ All energy in φ field implies homogeneity of φ over L_P

$$\boxed{S = S_p \exp\left[\frac{2\pi}{M^2} \left(\psi_p^2 - \psi^2 \right) \right]} \qquad (4)$$

(3) in (1) : $\boxed{\dot{\psi} = - \frac{mM}{\sqrt{12\pi}}}$

Evolution ψ



- since $H \propto \psi$, see (2), eventually weak damping limit is reached $\rightarrow$ oscillations
- coupling with other (weak) quantum fields becomes important $\rightarrow$ creation of matter $\rightarrow$ begin of hot Big Bang
- Inflation <u>creates</u> homogeneous, expanding, hot, flat FRW universe

Evolution S

<u>small t</u> : $S = S_p \exp\left[\frac{2\pi}{M^2} \left\{ \psi_p^2 - (\psi_p - \dot{\psi} t)^2 \right\} \right]$

$$= S_p \exp\Big[\underbrace{\frac{4\pi \psi_p \dot{\psi}}{M^2}}_{\text{constant}} t \Big]$$

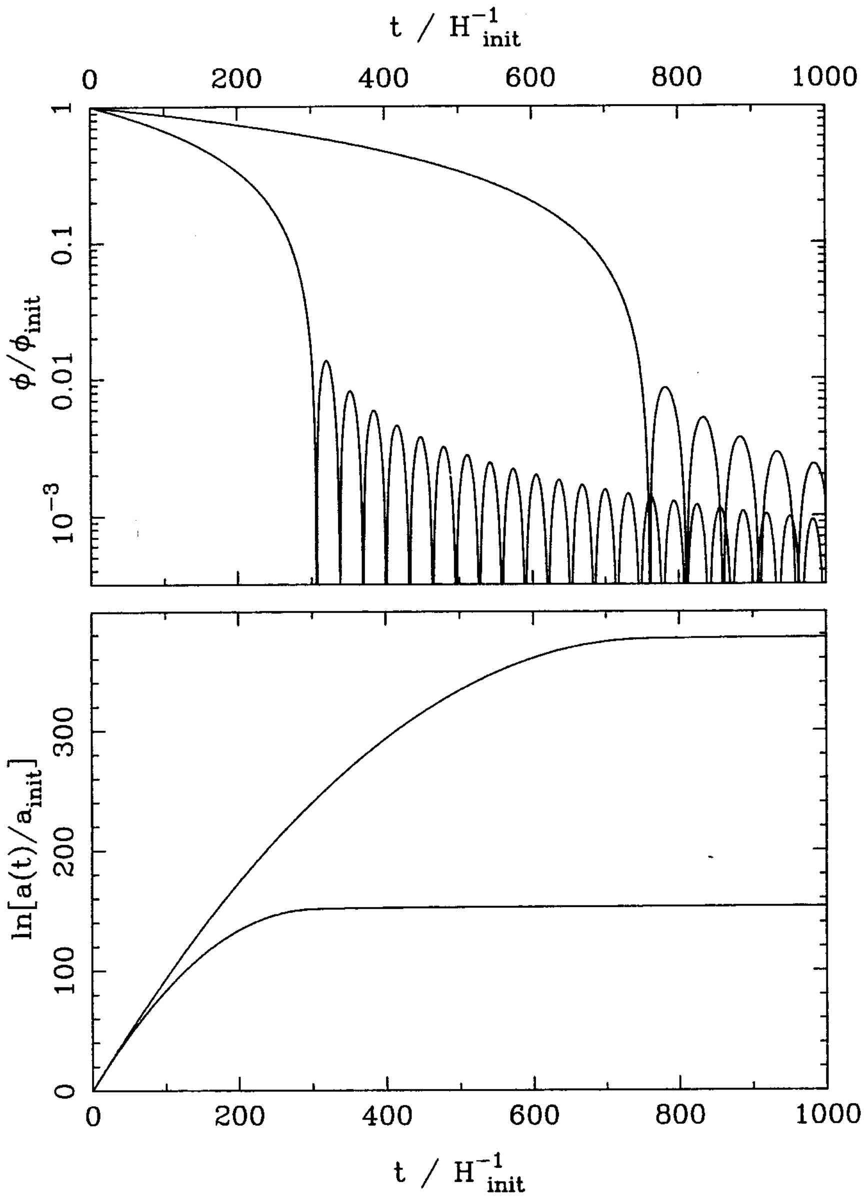
$\therefore$ Exponential Expansion

Large t

$$S = S_p \exp\left[\frac{2\pi}{M^2} \left(\phi_p^{\,2} - \phi^2 \right) \right]$$

$\phi^2 \simeq 0$ at end of inflation

$$\therefore \quad \frac{S_e}{S_p} \sim \exp\left[2\pi \frac{M^2}{m^2} \right] = \text{HUGE}$$



- Many inflation scenarios, none wholly acceptable

- this one called chaotic inflation, see Linde, Phys. Today, Sept '87, p. 61.

- conceptual picture $\Rightarrow$

- pro & cons

++ all energy in φ-field $\to$ dynamics φ probably reasonably described by equation for free particle.
No speculative particle physics needed!

-- classical reasoning right at $t = t_p$!
does Quantumgrav. permit instability of the vacuum ?

- what drives inflation ?

• φ field does not scale as S^{-3} like matter

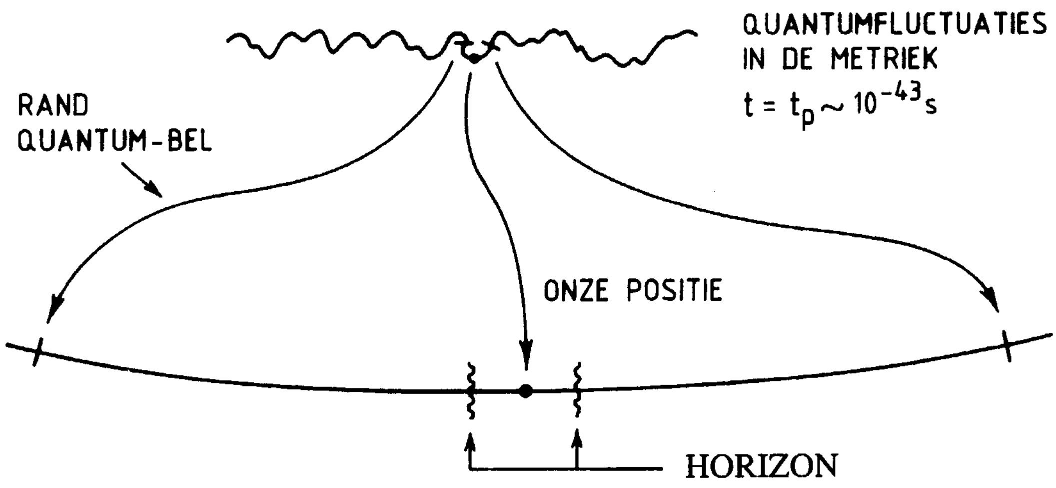
• Evolution S driven by equivalent P, ρ :

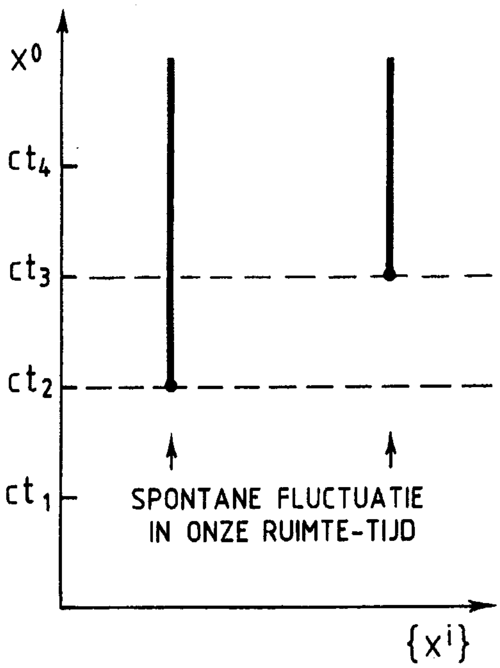
$$\rho = \tfrac{1}{2}\dot{\varphi}^2 + \tfrac{1}{2}m^2 \cdot \varphi^2 \simeq \tfrac{1}{2}m^2\varphi^2$$

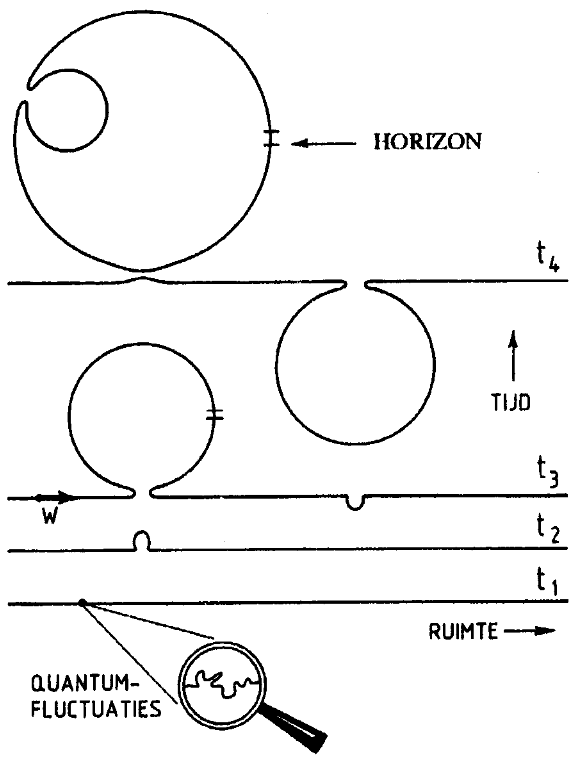
$$P = \tfrac{1}{2}\dot{\varphi}^2 - \tfrac{1}{2}m^2\varphi^2 \simeq -\tfrac{1}{2}m^2\varphi^2$$

vacuum with huge cosmol. constant Λ (:) $m^2\varphi^2$
huge antigravity from $P < 0$

• Global energy conservation does not exist in GR !!


QUANTUMFLUCTUATIES
IN DE METRIEK
$t = t_p \sim 10^{-43}$s
RAND
QUANTUM-BEL
ONZE POSITIE
HORIZON


x^0
ct_4
ct_3
ct_2
ct_1
SPONTANE FLUCTUATIE
IN ONZE RUIMTE-TIJD
$\{x^i\}$


HORIZON
t_4
TIJD
t_3
W
t_2
t_1
RUIMTE
QUANTUM-
FLUCTUATIES

THE BIG QUESTIONS

Observational

where are the optically dark baryons?

structure formation

relic $\nu\bar{\nu}$ $(T = 1.95\,K)$

Particle physics

origin $m\bar{m}$ asymmetry

nature nonbaryonic dark matter

properties quark-gluon plasma

Quantum gravity

origin Λ ?

acceptable inflation theory

Cosmologists have a great phantasy:

universe (>> our visible universe)
originates from tiny quantum fluctuation !?
true or not ? only future will tell