From Newtonian to Metric-affine Gravity

Recent developments and experiments

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Overview

- Basics of Newtonian gravity
- General relativity: Field equations & first modifications
- The gauge principle
- Metric-affine (gauge) theory of gravity
- Torsion + gravitational birefringence
- New tests using magnetic white dwarfs

Newtonian gravitation

1687: "Philosophia naturalis principia mathematica"

Important step towards an unification of Physics: Galilei + Kepler

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = - G \sum_{j=1, j \neq i}^{N} \frac{m_i m_j (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^3} \qquad (1)$$

$$G = (6,6726 \pm 0,0005) \cdot 10^{-11} \frac{m^3}{kg\, s^2}$$ Gravitational Constant

$$\phi(\vec{r}) = - G \sum_j \frac{m_j}{(\vec{r} - \vec{r}_j)} = - G \int_V d^3_{r'} \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \qquad (2)$$

$$\Rightarrow \quad m_I \frac{d^2 \vec{r}}{dt^2} = - m_G \vec{\nabla} \phi(\vec{r}) \qquad (3)$$ Equation of motion

$$\Rightarrow \quad \Delta \phi(\vec{r}) = 4 \pi G \rho(\vec{r})$$ Field equation in Newtonian gravitation

Basic problem: Action at a distance theory!

Analogous: Electrostatic Field equation

$$\Delta \phi_e = -4\pi \rho_e$$

electrostatic potential ($\Delta \phi_e$) — charge density (ρ_e)

Electrostatic $\longrightarrow$ Electrodynamic

$$\Delta \longrightarrow \Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$$

i.e. variations propagate with a <u>finite</u> speed c!

$$\rho_e \longrightarrow (\rho_e c, \rho_e v^i) = (j^\alpha)$$

$$\phi_e \longrightarrow (\phi_e, A^i) = (A^\alpha)$$

$$\Rightarrow \Delta \phi_e = -4\pi \rho_e \longrightarrow \Box A^\alpha = \frac{4\pi}{c} j^\alpha$$ Maxwell equations

Back to gravity:

$$\Delta \phi = 4\pi G \rho \longrightarrow \Box g^{\alpha\beta} \sim G\, T^{\alpha\beta}$$

General relativity

$$\Box g^{\alpha\beta} = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

Require:

1. $T^{\alpha\beta}$ is a Riemann Tensor
2. $T^{\alpha\beta}$ contains only first + second derivatives of $g^{\alpha\beta}$ (KISS)
3. $T^{\alpha\beta} = T^{\beta\alpha}$
4. Newtonian limit for weak fields

$$\Rightarrow \qquad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Field equation of General relativity

$$R_{\mu\nu} = R^{\kappa}{}_{\mu\kappa\nu} = g^{\lambda\kappa} R_{\mu\lambda\kappa\nu} \, , \quad R = R^{\mu}{}_{\mu} = g^{\mu\nu} R_{\mu\nu}$$

$$R_{\kappa\mu\lambda\nu} = \frac{1}{2}\left(\frac{\partial^2 g_{\kappa\lambda}}{\partial x^{\mu}\partial x^{\nu}} + \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa}\partial x^{\lambda}} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^{\kappa}\partial x^{\nu}} - \frac{\partial^2 g_{\kappa\nu}}{\partial x^{\mu}\partial x^{\lambda}}\right)$$

$$+ g_{\alpha\beta}\left(\Gamma^{\alpha}_{\lambda\kappa}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\nu\kappa}\Gamma^{\beta}_{\mu\nu}\right)$$

Curvature tensor

$$\left[\Gamma^{\kappa}_{\lambda\mu} = \frac{1}{2} g^{\kappa\nu}\left(\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}}\right)\right]$$

Are these field equations unique?

Fixed by requirements (1) - (4), but...

We can drop (2) and allow for terms, linear in $g_{\mu\nu}$

$$\Rightarrow \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = - \frac{8\pi G}{c^4} T_{\mu\nu}$$

Newtonian limit: $\Delta\phi = 4\pi G \rho - \frac{1}{2} c^2 \Lambda$

$= 4\pi G (\rho - \rho_{VAC})$

$\rho_{VAC} = \frac{c^2}{8\pi G} \Lambda$ Vacuum Energy density

Λ has to be small to be in agreement with Newton within the Solar system

$\frac{1}{\sqrt{\Lambda}} \gg$ diameter of Solar system

e.g. $\Lambda^{-\frac{1}{2}} \approx 10^7 Lj \Rightarrow$ not relevant within Solar system $\lesssim 1Ld$

or Galaxy $\approx 10^5 Lj$

but important on cosmological scale

Λ: Cosmological constant

Basic criteria for the viability of a gravitation theory

1. It must be complete

   Kinematical relativity (Milne, 1948) : makes no gravitational red-shift prediction

2. It must be selfconsistent:
   (predictions for the outcome of every experiment must be unique)

   Various theories by Withrow & Morduch (1965)

   Different results for light propagation for light viewed as particles and light viewed as waves.

3. It must be relativistic
   (reduce to Special Relativity as gravity is "turned off")

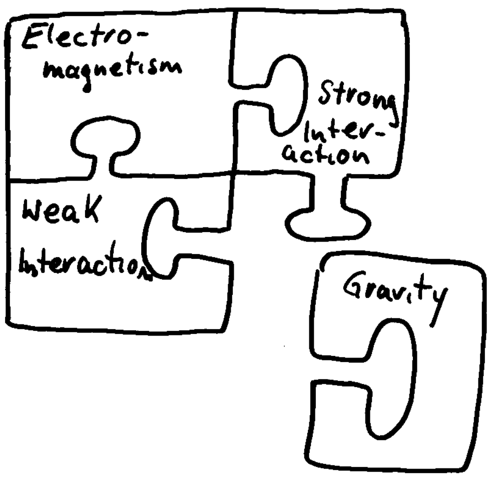
   Newtonian gravitation theory

4 classical tests support General Relativity (at least)

- Light deflection (1919)
- Perihelion shift of Mercury
- Gravitational red-shift (test of the EEP)
- Radar echo delay

... sofar everything is fine, but . . .

... General Relativity is a purely classical theory!



Where do we have to modify GR without experimental hints?

→ The gauge principle

The gauge principle

System described by $\psi(x)$ (e.g. electron wavefunction)

actually "observable": $|\psi(x)|^2$ probabilistic interpretation

Simplest example: $U(1)$ - Symmetry

The Dirac-Field

$$\mathcal{L}_0 = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi \qquad (1)$$

now:

$$\begin{aligned} \psi(x) &\to \psi'(x) = e^{-i\alpha(x)}\,\psi(x) \\ \bar{\psi}(x) &\to \bar{\psi}'(x) = e^{+i\alpha(x)}\,\bar{\psi}(x) \end{aligned} \qquad (2)$$

clear: $|\psi'(x)|^2 = |e^{-i\alpha(x)}|^2\,|\psi(x)|^2 = |\psi(x)|^2$

$\Rightarrow \mathcal{L}_0$ has to be valid also for $\psi'(x)$, but ...

$$\partial_\mu \psi(x) \to (\partial_\mu \psi)'(x) = e^{-i\alpha(x)}(\partial_\mu \psi)(x) - \underbrace{(i\partial_\mu \alpha(x))\,e^{-i\alpha(x)}\,\psi(x)}_{\text{additional term}}$$

Asymmetry!

$$\Rightarrow \delta\mathcal{L}_0 = \bar{\psi}\gamma^\mu\psi\,(\partial_\mu \alpha)$$

(1) is not invariant under the transformation (2)

Solution: Introducing a new gauge field A_μ

Define covariant derivative: $D_\mu \psi(x) \equiv (\partial_\mu - ie A_\mu(x)) \psi(x)$

with $\quad A_\mu \to A'_\mu = A_\mu + \partial_\mu f \quad ; \quad f = -\frac{1}{e} \alpha(x)$

$$\Rightarrow (D_\mu \psi(x))' = (\partial_\mu - ie A'_\mu(x)) \psi'(x)$$

$$= i(\partial_\mu \alpha(x)) e^{-i\alpha(x)} \psi(x) + e^{-i\alpha(x)} \partial_\mu \psi(x)$$

$$- i(e A_\mu(x) + \partial_\mu \alpha(x)) e^{-i\alpha(x)} \psi(x)$$

$$= e^{-i\alpha(x)} (\partial_\mu - ie A_\mu(x)) \psi(x)$$

$$= e^{-i\alpha(x)} D_\mu \psi(x)$$

No asymmetry!

Note: A_μ is massless since $\frac{1}{2} m A^\mu A_\mu$ is not invariant

$\Rightarrow$ Photonfield

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{INT}$, $\mathcal{L}_{INT} = -j^\mu A_\mu$ invariant

Invariance of the Lagrangian under local symmetry transformations requires the existence of additional gauge fields!

The gauge principle is the cornerstone of the Standardmodel

$$SU(3) \times \underbrace{SU(2) \times U(1)}$$

↑ Strong Interaction — Electroweak Interaction (↖)

What about Gravity?

Weyl (1919)

Utiyama, Sciama & Kibble (1961):

General relativity as a gauge theory of the Poincaré-group

Poincaré-group: $\vec{x} \rightarrow \vec{x}' = \Lambda \vec{x} + \vec{a}$

(Λ ↑ Lorentz transformation)

Currently most promising: Metric-affine gauge theory of gravity (Hehl et al. 1994)

Symmetry group: $\xi \rightarrow \xi' = \Lambda \xi + \tau$

$\xi, \tau \in \mathbb{R}^n$, $\Lambda \in GL(n, \mathbb{R})$ General linear group

New gravitational gauge fields:

- Torsion: Influence of the spin on the structure of spacetime
- Nonmetricity: Deformation of length and angle standards during parallel transport

Problem:

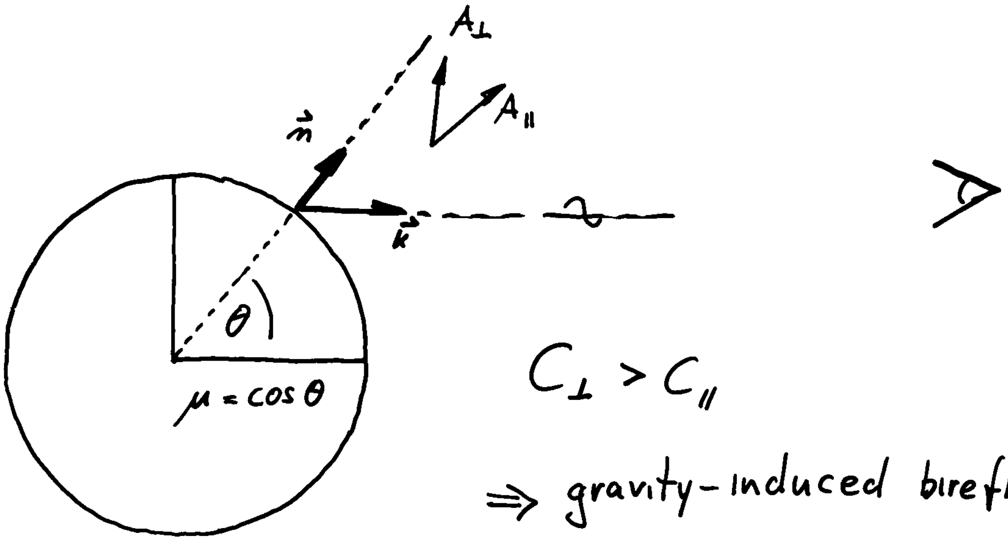
Currently __no__ experimental evidence for the existence of Torsion!

Rest of the talk is therefore devoted to the following questions:

1. How does Torsion possibly interact with the electromagnetic field?
2. What are the theoretical and experimental consequences?
3. Where (and how) do we have to observe?

Ansatz: $\mathcal{L} = k^2 * (T_\alpha \wedge F)\, T^\alpha \wedge F$ (Solanki, Preuss, Haugan 2003)

- Conservation of electric charge
- So far unique in metric-affine gravity
- Predicts gravitational-birefringence (violates the EEP)



$C_\perp > C_\parallel$

$\Rightarrow$ gravity-induced birefringence!

Observable effects of gravity-induced birefringence

hase shift between orthogonal polarization components

$$\Delta \phi_{MAG}(\mu) = \sqrt{\frac{2}{3}} \; \frac{k^2 \, 2\pi M_*}{\lambda R^3} \; \frac{(\mu+2)(\mu-1)}{\mu+1}$$

- What is our objective?
  Setting strong limits on k^2 and decide about the physical relevance of this approach.

- What are we looking for?
  Depolarization effects

- Where do we observe?
  Massive objects with strong gravitational + magnetic fields

$\Longrightarrow$ Magnetic white dwarfs

Magnetic white dwarfs (MWDs)

Basics:

- approx. 65 white dwarfs are classified as MWD ($\approx 5\%$ of all)
- Field strength $3 \cdot 10^4\,G \sim 10^9\,G$
- Dipolar (and often decentered) fields
- Circular polarization up to $\sim 20\%$!! (RE J0317-853)

IDEA

MAG

$\Downarrow$

$k^2 \neq 0$

$\Downarrow$

different $\Delta\phi$ from all parts of extended source

$\Downarrow$

depolarization of combined polarization from extended source

Observation of polarization from extended source $\neq 0$

LIMIT ON k^2

RE J0317-853

- Discovered in 1995 during ROSAT all-sky survey
- very remarkable object:

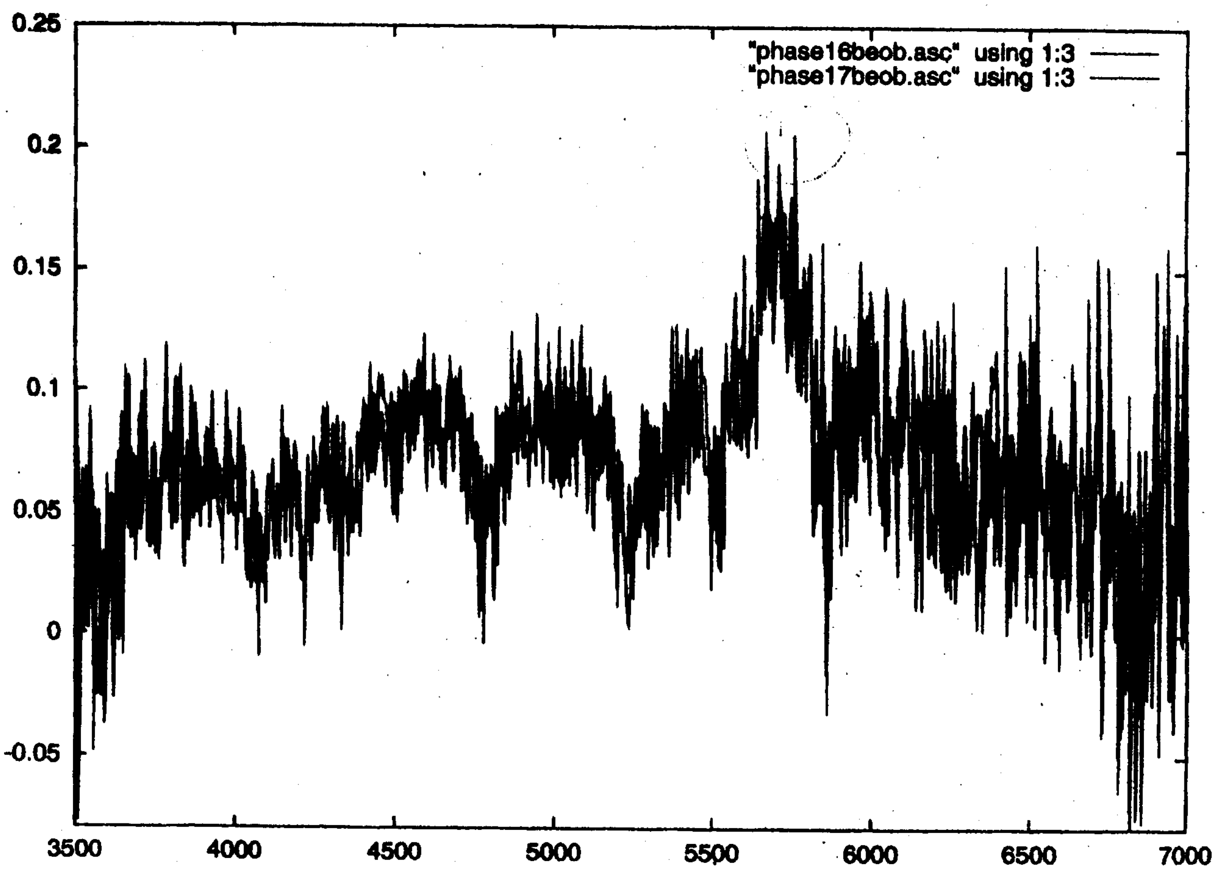
$$P_{ROT} = 725 \text{ sec}$$

$$M_* = 1,35\ M_\odot$$

$$R_* = 0,0035\ R_\odot$$
$$= 0,38\ R_{EARTH}$$

$$B = 1,4 \cdot 10^8\ G \sim 7,3 \cdot 10^8\ G$$

$$V_{MAX} \approx 20\%\ !$$



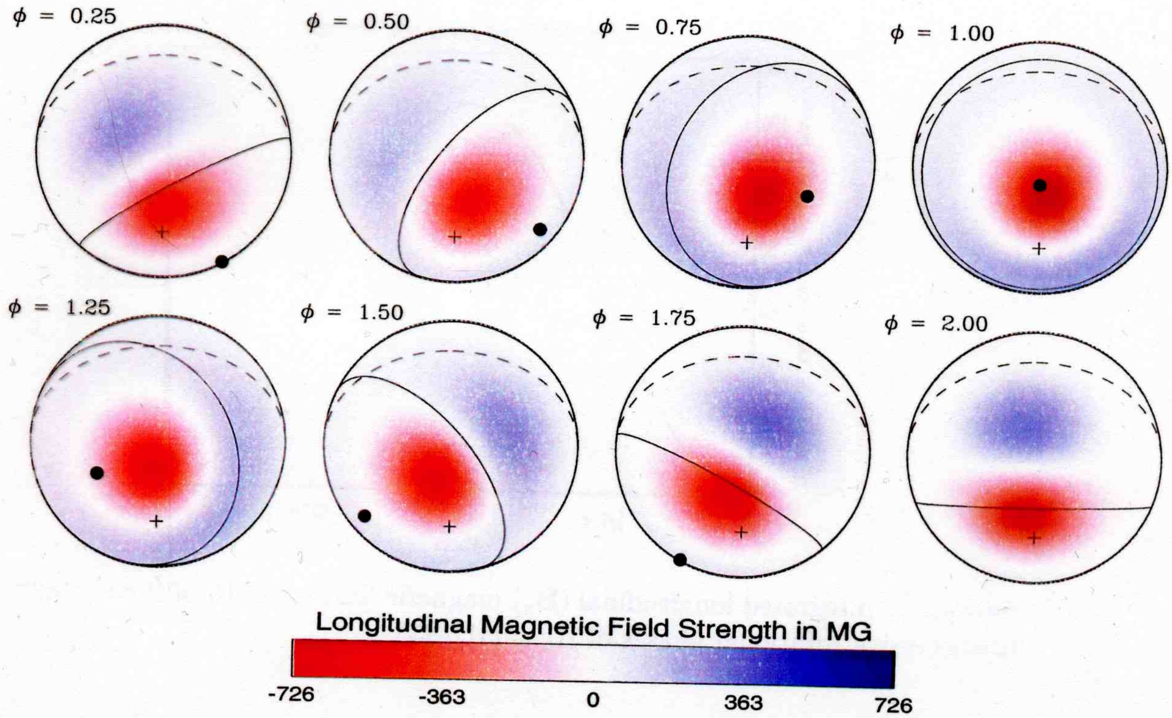
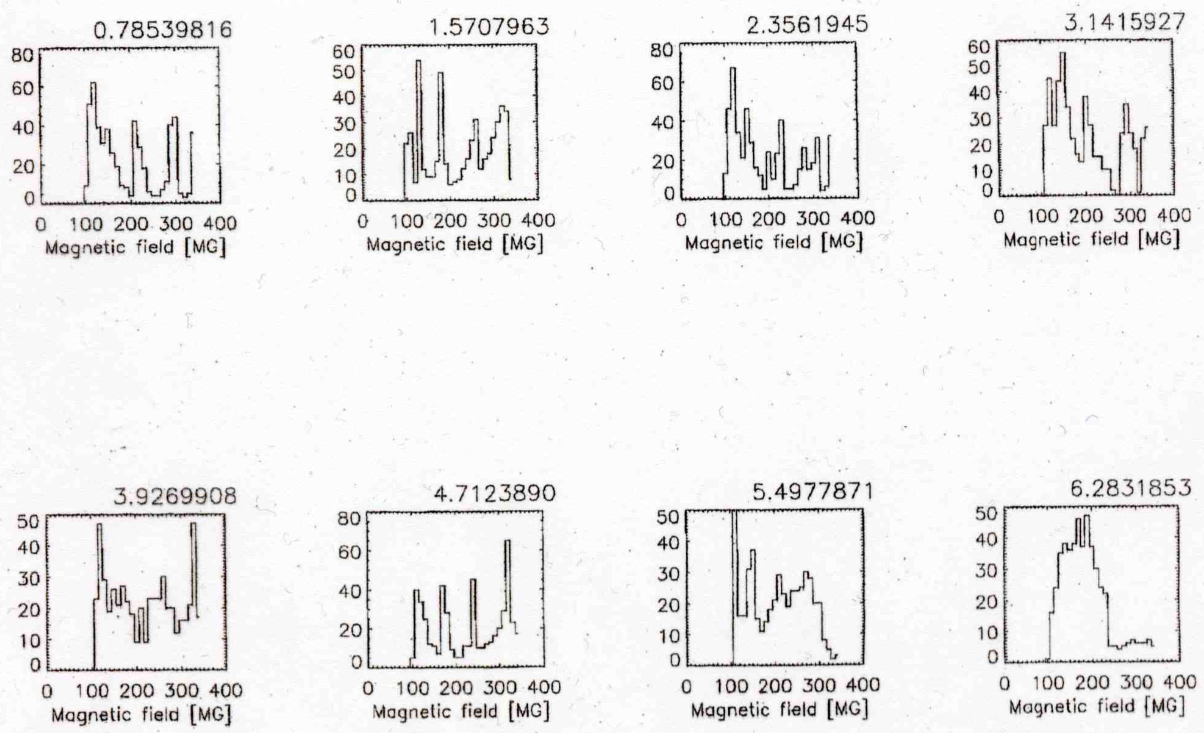


Figure 3.5: Successive rotational phases of RE J0317-853 in steps of 0.25π, beginning with $\phi = 0.25\pi$ (top left) to $\phi = 2\pi$ (bottom right). The dashed line markes the stellar equator whereas the solid line shows the projection of the magnetic equator on the stellar surface. The cross marks the position of the rotation axis and the dot the position of the magnetic pole.



Magnetic field distribution between 140 MG and 200 MG at phase with strong polarization

$\Downarrow$

Extreme assumption:
All field vectors point towards observer

$\Downarrow$

Maximum possible polarization 26.5 %

$\Downarrow$

Observed: ~17%–20%

$$\hbar^2 \lesssim (19\,m)^2$$

all magnetic fields < 530 MG

$\Downarrow$

$$\hbar^2 \lesssim (30.5\,m)^2$$

Compare: Sun: $\hbar^2 \leq (1950\,m)^2$
(Solanki, Preuss, Haugan 2003)

Summary

- General relativity is a valid but incomplete theory
- Gauge principle as a guiding-line
- Metric-affine gravity best alternative for GR
- Coupling between Torsion + Electromagnetism leads to gravitational birefringence
- Strong limits on the relevant coupling constant k^2 by using magnetic white dwarfs

To do

- Gravitational lensing + birefringence
- Torsion in the early universe