International Max Planck Research School
on Physical Processes in the Solar System and Beyond
at the Universities of Braunschweig and Göttingen

Solar System School

# HYDRODYNAMICS

**Antonio Ferriz Mas, University of Vigo**

**21 – 25 July 2008**

1. **Kinematics of the continuum. Spatial and material descriptions.**
   - 1.1. Spatial (eulerian) and material (lagrangian) descriptions of the motion. Material derivative. Velocity and acceleration. Trajectories and streamlines.
   - 1.2. Deformation and vorticity tensors. Physical interpretation.
   - 1.3. Reynolds' transport theorem.
2. **Fundamental equations in Continuum Mechanics.**
   - 2.1. Conservation of mass: continuity equation.
   - 2.2. Long-range (volume) forces and contact (surface) forces. Stress tensor.
   - 2.3. Momentum balance: equation of motion. Mechanical energy balance.
   - 2.4. Angular momentum balance: symmetry of the stress tensor.
   - 2.5. Conservation of energy and first principle of Thermodynamics.
   - 2.6. Constitutive relations.
3. **Viscous fluids. Navier-Stokes equation.**
   - 3.1. Hydrostatic pressure. Ideal fluid model. Euler's equation.
   - 3.2. Stress tensor for a linearly viscous fluid. Newtonian fluid model. Coefficients of viscosity. Derivation of Navier-Stokes' equation.
   - 3.3. Boundary conditions.
   - 3.4. Scale analysis and dimensionless numbers.
4. **Energy equation for a Newtonian fluid.**
   - 4.1. Second principle of Thermodynamics. Energy equation in the entropic representation. Concepts of adiabatic, isentropic and homoentropic motions.
   - 4.2. Heat conduction. Entropy sources.
   - 4.3. Alternative forms of expressing the energy equation.
5. **The hydrodynamic equations in conservation form.**
   - 5.1. Momentum equation in conservation form. The momentum density flux tensor.
   - 5.2. Energy equation in conservation form. The energy density flux vector.
   - 5.3. Derivation of the jump relations across a discontinuity. Tangential discontinuities and shock fronts. Rankine-Hugoniot relations.
6. **Circulation and vorticity.**
   - 6.1. Circulation and vorticity. Vortex tubes. Some kinematic results.
   - 6.2. Theorems of Kelvin and Helmholtz for ideal fluids.
   - 6.3. Navier-Stokes' equation in terms of the vorticity. 2-D results.
   - 6.4. Crocco's equation and Bernoulli's theorems.

# HYDRODYNAMICS



**Bulletin of exercises n°1:** Kinematic aspects in Continuum Mechanics. The Jacobian and its geometrical interpretation. Equation of continuity.

The motion of a fluid is given by a function

$$\mathbf{x} = \mathbf{X}(\mathbf{a}, t)\,, \tag{1}$$

where $\mathbf{a}$ is the position vector at $t = 0$ of the fluid element which is at position $\mathbf{x}$ at time $t$: $\mathbf{X}(\mathbf{a}, 0) = \mathbf{a}$.

For each fixed $t$, (1) defines an invertible transformation of the continuum onto itself. The Jacobian determinant

$$J\,(\mathbf{a}, t) \stackrel{\text{def}}{=} \det \left(\frac{\partial \mathbf{X}}{\partial \mathbf{a}}\right)_t = \det D\mathbf{X}(\mathbf{a}, t)$$

is, therefore, always different from zero.

## 1. Streamlines and trajectories.

Prove that for a time-independent velocity field $\mathbf{v}(\mathbf{x})$ the trajectories and the streamlines coincide geometrically.

## 2. Euler's identity.

For the proof of Reynolds' transport theorem use is made of the so-called *Euler's identity*,

$$\left(\frac{\partial J}{\partial t}\right)_{\mathbf{a}} = J \operatorname{div} \mathbf{v}\,,$$

where $J$ is the Jacobian determinant of the transformation $\mathbf{x} = \mathbf{X}(\mathbf{a}, t)$ and $\mathbf{v}(\mathbf{x}, t)$ is the velocity field.

Prove Euler's identity.

## 3. Incompressible flows.

A flow is said to be incompressible if the volume of any arbitrary portion of the fluid remains constant in time; i.e., if for any arbitrary $\Omega_{\rm t}$ it holds that

$$\frac{d}{dt}\int_{\Omega_{\rm t}} 1 \;=\; 0\,.$$

- Prove that a flow is incompressible **if and only if** the Jacobian $J$ is equal 1 at all times.
- Prove that a flow is incompressible **if and only if** $D\rho/Dt \equiv 0$.

## 4. Continuity equation in the material representation.

Prove that the assumption of mass conservation yiels the following relation,

$$\rho(\mathbf{a}, t)\, J(\mathbf{a}, t) = \rho(\mathbf{a}, 0)\,, \tag{2}$$

which can be called "the equation of continuity in the material (or *Lagrangian*) representation."

Check that you come to the same result if you start from the *customary form* of the continuity equation and apply Euler's identity.

**Bulletin of exercises n° 2:** Constitutive relations for a linearly viscous fluid.

In order to obtain the form of the stress tensor for *linearly viscous fluids* the following assumptions are made:

$(i)$ The stress tensor at any point $\mathbf{x}$ of the medium depends on the thermodynamic state and on the gradient-of-velocity tensor at this point.

$(ii)$ When the fluid is at rest (or there are no gradients of velocity), the constitutive relation for the stress tensor must reduce to that of hydrostatics.

From $(i)$ and $(ii)$ it follows that the constitutive relation for $\sigma_{ij}$ must be of the form

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}(v_{r,s}, \rho, T)\,, \qquad \text{with} \qquad \tau_{ij}(0, \rho, T) = 0\,, \tag{1}$$

where we have chosen the variables $\rho, T$ to represent the thermodynamic state of the fluid. The coefficient $p$ which multiplies $\delta_{ij}$ is the *thermodynamic* pressure, which coincides with the *mechanical pressure* when the fluid is at rest (i.e., with the *hydrostatic* pressure).

In this excercise the form of the nine functions $\tau_{ij}$ is deduced.

**1.** It is required that the relationship between $\hat{\boldsymbol{\sigma}}$ and $\nabla\mathbf{v}$ be linear (this is the simplest extension of the constitutive relation $\hat{\boldsymbol{\sigma}} = -p\hat{\boldsymbol{I}}$ which includes the dependence of $\hat{\boldsymbol{\sigma}}$ with $\nabla\mathbf{v}$ and is compatible with "Navier's hypothesis" for viscosity).

Show that this linear relationship along with the symmetry of $\hat{\boldsymbol{\sigma}}$ imply that (1) can be written in the form

$$\sigma_{ij} = -p\delta_{ij} + C_{ijrs}v_{r,s}\,, \qquad \text{with} \qquad C_{ijrs} = C_{jirs}\,, \tag{2}$$

where $\hat{\boldsymbol{C}} = (C_{ijrs})$ is a fourth-order tensor whose components are functions of $\rho$ and $T$. How many components has $\hat{\boldsymbol{C}}$?

## Newtonian fluids

**2.** Prove that if, further, isotropy is assumed (i.e., that there are no preferred directions in space), the number of independent elements of the tensor $\hat{\boldsymbol{C}}$ reduces to only two. Write the general form of $C_{ijrs}$.

*Hint:* The most general isotropic fourth-order tensor has the form

$$A_{ijrs} = \lambda\delta_{ij}\delta_{rs} + \mu(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr}) + \eta(\delta_{ir}\delta_{js} - \delta_{is}\delta_{jr})\,. \tag{3}$$

**3.** Prove that $C_{ijrs}v_{r,s} = C_{ijrs}D_{rs}$, where $\hat{\boldsymbol{D}}$ is the symmetric part of $\nabla\mathbf{v}$; that is, the antisymmetric part of $\nabla\mathbf{v}$ is not involved in the constitutive relations. Discuss the physical interpretation of this fact.

**4.** Write the resulting constitutive relations (which describe the model called "Newtonian fluid"). Obtain the relation existing between the thermodynamic and the mechanical pressures.

[NOTE: The mechanical pressure at $\mathbf{x}$ is defined as minus the mean value of the normal stresses at $\mathbf{x}$].

# HYDRODYNAMICS



**Bulletin of exercises n° 3:** Basic equations in conservation form.

## 1. Interpretation of the continuity equation.

The continuity equation can be written in the form:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho\,\mathbf{v}) = 0\,. \tag{1}$$

Integrate Eq. (1) in a **fixed region** $\mathcal{R}$ in 3-D space with boundary $\partial\mathcal{R}$ and apply Gauß' theorem. Interpret the result and justify the name *mass flow vector* for the vector quantity $\rho\mathbf{v}$.

Note that this interpretation of the continuity equation is equally valid for any other equation that can be formally written in the form (1), viz.

$$\frac{\partial Q}{\partial t} + \mathrm{div}\,\mathbf{J}_Q = 0\,, \tag{2}$$

where $Q$ is the *density* of a physical quantity (i.e., per cm$^3$) and $\mathbf{J}_Q$ is its corresponding flux density (i.e., per cm$^2$).

The momentum equation (Euler's equation) and the energy equation can also be written in this way. An evolution equation expressed in the form (2) is said to be written in *conservation form*. This form is particularly useful in the numerical integration of hydrodynamic and magnetohydrodynamic problems. It is also especially appropriate for the interpretation of energy and linear momentum fluxes in problems of wave propagation and for the derivation of the relations holding across flow discontinuities (in particular, shock fronts).

## 2. The momentum equation in conservation form.

Show that if the long-range force per unit volume derives from a scalar potential $\Phi$, then the equation of motion for a fluid can be written in the form

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \mathrm{div}\,\left(\hat{\mathbf{\Pi}} + \Phi\,\hat{\boldsymbol{I}}\right) = \mathbf{0}\,, \tag{3}$$

where $\hat{\mathbf{\Pi}} \stackrel{\mathrm{def}}{=} \rho\mathbf{v}\otimes\mathbf{v} - \hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{I}}$ is the unit tensor.

If the flow is inviscid, then Eq. (3) can be written

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \mathrm{div}\,\left[\hat{\mathbf{\Pi}} + (p+\Phi)\,\hat{\boldsymbol{I}}\right] = \mathbf{0}\,, \tag{3a}$$

where now $\hat{\mathbf{\Pi}} \stackrel{\mathrm{def}}{=} \rho\mathbf{v}\otimes\mathbf{v}$.

In order to understand the physical meaning of the tensor $\hat{\mathbf{\Pi}}$, integrate Eq. (3) in a fixed region $\mathcal{R}$ bounded by a regular surface $\partial\mathcal{R}$.

## 3. The energy equation in conservation form.

Under the assumption that the long-range forces per unit mass derive from a time-independent scalar potential $\Psi$, show that the energy equation (which is the differential expression of the principle of energy conservation) can be written in the form

$$\frac{\partial \mathcal{E}}{\partial t} + \mathrm{div}\,\mathbf{J}_{\mathcal{E}} = 0\,, \tag{4}$$

where $\mathcal{E} = \rho\,(\epsilon + \|\mathbf{v}\|^2/2 + \Psi)$ is the total energy density (i.e, per cm$^3$) and $\mathbf{J}_{\mathcal{E}}$ is its corresponding flux density (i.e, per cm$^2$).

In order to understand the physical meaning of the flux vector $\mathbf{J}_{\mathcal{E}}$, integrate Eq. (4) in a fixed region $\mathcal{R}$ bounded by a regular surface $\partial\mathcal{R}$.

Indication: Employ the original expression of Reynolds' transport theorem (i.e., without making use of the continuity equation).

# HYDRODYNAMICS

International Research School. Max-Planck-Institut für Sonnensystemforschung (Lindau). 21 – 25 July 2008

**Bulletin of exercises n° 4:** Energy equation for a compressible flow.

**1. Energy equation in the representation** $\mathbb{G}(p, T)$: Assume that the thermodynamic state of a fluid of constant chemical composition can be specified through the variables $(p, T)$. Without assuming any further thermodynamic restriction (such as considering an *ideal gas*), show that the differential expression of the first principle of Thermodynamics is given by

$$\rho\, c_p \frac{D\,T}{D\,t} - \alpha T \frac{D\,p}{D\,t} = -p \operatorname{div} \mathbf{v} + \Phi_{\mathrm{V}} - \operatorname{div} \boldsymbol{\mathcal{F}}\,, \tag{1}$$

where $\Phi_{\mathrm{V}}$ is the viscous dissipation function, $\alpha$ is the thermal expansion coefficient and $c_p$ is the specific heat at constant pressure.

In many cases, the thermodynamic behaviour of a gas can be quite well approximated by the *ideal gas model*. For instance, air (except when changes of state occur) or the stellar plasma in most main-sequence stars. Which particular form does equation (1) take if the fluid is assumed to be an ideal gas? [Note: You may need to refresh your notions of elementary Thermodynamics. As a suggestion, *Thermodynamics and an Introduction to Thermostatistics*, H. B. Callen, Wiley, 2nd edition, 1985].

**2. Adiabatic processes**: Consider the local expression of the first principle of Thermodynamics in terms of the variables pressure and temperature [Eq. (1)] and assume that the heat flux vector satisfies *Fourier's law* for heat conduction.

Call $L$ the characteristic length of variation of temperature, $\tau$ the characteristic time for the evolution of $T$ and $\kappa_T$ the coefficient of thermal conductivity.

Show that, in orders of magnitude, the ratio between the heat diffusion term and the term expressing temperature variation is equal to the ratio between the characteristic evolution time for $T$ and the characteristic time for heat conduction (call it $\tau_c$).

Justify why if $\tau \ll \tau_c$ we can say that the process is adiabatic.

**3. Physical interpretation of the quantity enthalpy:** The physical quantity *enthalpy* is useful in a variety of situations in compressible Hydrodynamics. It appears in a natural way, for instance, in studying energy propagation by waves or in the Rankine-Hugoniot jump relations across a shock front.

In order to better understand its physical meaning, write the first principle of Thermodynamics replacing the specific internal energy $\epsilon$ by the specific enthalpy $h$ (defined as $h \stackrel{\text{def}}{=} \epsilon + p/\rho$) and show that it is equivalent to

$$\rho \frac{D\,h}{D\,t} - \frac{D\,p}{D\,t} = \Phi_{\mathrm{V}} - \operatorname{div} \boldsymbol{\mathcal{F}}\,. \tag{2}$$

From Eq. (2) show that if a mass of fluid is undergoing a process at uniform pressure $p$, then the rate of change of the total enthalpy of that mass of fluid satisfies the inequality

$$\dot{H}(\Omega_t) \geq Q\,(\partial\Omega_t)\,, \tag{3}$$

where $\Omega_t$ is the region occupied by the mass of fluid at time $t$ and $Q(\partial\Omega_t)$ is the heat per unit time being exchanged between the fluid mass and its surrounding through its boundary.

Under which conditions would the equal sign hold in inequality (3)? From this result, explain the physical meaning of the thermodynamic quantity "enthalpy."

# HYDRODYNAMICS

International Research School. Max-Planck-Institut für Sonnensystemforschung (Lindau). 21 – 25 July 2008

## Bulletin of exercises n° 6: Vorticity.

Circulation and vorticity are two quantities that measure the degree of rotation in a fluid. Circulation, a scalar quantity, is an integral measure of rotation, while vorticity, a vector quantity, yields a local measure of the degree of rotation in the neighborhood of a fluid element.

### 1. Kinematic identities.

In many cases it is convenient to describe the flow of a fluid in terms of the vorticity field, $\boldsymbol{\omega} = \mathbf{rot}\,\mathbf{v}$. In order to obtain an equation of motion based on vorticity, it is especially useful to make use of a number of *kinematic identities* involving the vorticity field.

Prove the following vector identities:

$$\mathbf{q} \stackrel{\text{def}}{=} \frac{D\,\mathbf{v}}{D\,t} = \frac{\partial \mathbf{v}}{\partial t} + \boldsymbol{\omega} \wedge \mathbf{v} + \frac{1}{2}\mathbf{grad}\,||\mathbf{v}||^2\,, \tag{i}$$

$$\mathbf{rot}\,\mathbf{q} = \frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{rot}\,(\boldsymbol{\omega} \wedge \mathbf{v})\,, \tag{ii}$$

$$\mathbf{rot}\,\mathbf{q} = \frac{D\,\boldsymbol{\omega}}{D\,t} - (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \boldsymbol{\omega}\,\mathrm{div}\,\mathbf{v}\,. \tag{iii}$$

Combining identity (iii) with the continuity equation, the so-called *Beltrami identity* is obtained:

$$\frac{D}{D\,t}\left(\frac{\boldsymbol{\omega}}{\rho}\right) = \left(\frac{\boldsymbol{\omega}}{\rho} \cdot \nabla\right)\mathbf{v} + \frac{1}{\rho}\mathbf{rot}\,\mathbf{q}\,. \tag{iv}$$

### 2. Transport theorem for a material curve:

Let $\boldsymbol{\alpha} = \boldsymbol{\alpha}(\xi, t)$ be the parametric expression of the material curve $\mathrm{C_t}$ and $\xi \in [\xi_1, \xi_2]$ the parameter (chosen in such a way that it *labels* each matter element along the curve in a unique way. Let $\mathbf{Q}(\mathbf{x}, t)$ be a vector quantity defined in the flow region and consider the line integral

$$\int_{\mathrm{C_t}} \mathbf{Q} \cdot d\,\boldsymbol{\alpha} \stackrel{\text{def}}{=} \int_a^b \mathbf{Q}[\boldsymbol{\alpha}(\xi, t)] \cdot \boldsymbol{\alpha}'(\xi, t)\,d\,\xi\,. \tag{1.1}$$

Show that the time derivative of (1.1) is

$$\frac{d}{dt}\int_{\mathrm{C_t}} \mathbf{Q} \cdot d\,\boldsymbol{\alpha} = \int_{\mathrm{C_t}} \frac{D\,\mathbf{Q}}{D\,t} \cdot d\,\boldsymbol{\alpha} + \int_a^b \mathbf{Q} \cdot [\,(\boldsymbol{\alpha}' \cdot \nabla)\,\mathbf{v}\,]\,d\,\xi\,, \tag{1.2}$$

where $\boldsymbol{\alpha}'$ is the partial derivative of $\boldsymbol{\alpha}$ with respect to the parameter $\xi$.

This result is the one-dimensional version (i.e., for a material curve) of Reynold's theorem.

**3. Circulation of the velocity:** The circulation of the velocity along the material circuit $\mathrm{C_t}$ is defined as:

$$\Gamma(t) = \oint_{\mathrm{C_t}} \mathbf{v} \cdot d\,\boldsymbol{\alpha} \stackrel{\text{def}}{=} \int_{\xi_1}^{\xi_2} \mathbf{v}[\boldsymbol{\alpha}(\xi, t)] \cdot \boldsymbol{\alpha}'(\xi, t)\,d\,\xi\,, \tag{2.1}$$

where $\boldsymbol{\alpha} = \boldsymbol{\alpha}(\xi, t)$ is the parametric expression of the curve and $\xi \in [\xi_1, \xi_2]$ is the parameter.

Show that the time derivative of the circulation is given by

$$\dot{\Gamma}(t) = \frac{d}{dt}\oint_{\mathrm{C_t}} \mathbf{v} \cdot d\,\boldsymbol{\alpha} = \oint_{\mathrm{C_t}} \frac{D\,\mathbf{v}}{D\,t} \cdot d\,\boldsymbol{\alpha}\,. \tag{2.2}$$

**Bulletin of exercises n° 6** (continuation): Vorticity.

## 4. Evolution equation for the kinematic helicity density.

Given a velocity field $\mathbf{v}(\mathbf{x}, t)$, the *kinematic helicity density* is $\mathbf{v} \cdot \boldsymbol{\omega}$ and the helicity of a material region $\Omega_{\rm t}$ is defined as

$$\mathcal{H}(\Omega_{\rm t}) = \int_{\Omega_{\rm t}} \mathbf{v} \cdot \boldsymbol{\omega} \, .$$

Consider a fluid flow in a barotropic medium (i.e., isopycnic surfaces coincide with isobaric surfaces) under conservative long-range forces and in a situation in which the effects of viscosity can be neglected.

Prove that, under these conditions, from the equations of continuity and motion the following equation can be derived, which governs the evolution of the *kinematic helicity density*, viz.

$$\frac{D}{D\,t}\left(\frac{\mathbf{v}\cdot\boldsymbol{\omega}}{\rho}\right) = \left(\frac{\boldsymbol{\omega}}{\rho}\cdot\nabla\right)(\mathbf{v}\cdot\mathbf{v} - \varphi) \,, \tag{3}$$

where $\varphi(\mathbf{x}, t)$ is a scalar funtion that must be determined.

## 5. Conservation of kinematic helicity.

Under the same conditions for which the theorems of Kelvin and Helmholtz hold, the kinematic helicity of a vortex tube is conserved.

- Let $\Omega_{\rm t}$ be a vortex tube. Prove that under the conditions of exercise **n°4** the kinematic helicity of a vortex tube is a constant of motion.
- How does the result depend on the funtion $\varphi$ (previous exercise)? Does this result hold for *any* material region $\Omega_{\rm t}$? Why?
- Why do we need to assume a *barotropic medium* in order to come to the result? Could we state the assumption “isopycnic surfaces coinciding with isobaric surfaces” in a different way that does not involve pressure and density?

International Max Planck Research School
on Physical Processes in the Solar System and Beyond
at the Universities of Braunschweig and Göttingen

# HYDRODYNAMICS

**Antonio Ferriz Mas, University of Vigo**

**31 January – 4 February 2005**

## Part I. Basic equations and concepts in Fluid Mechanics.

### 1. Kinematics of the continuum. Spatial and material descriptions.

1.1. Spatial (eulerian) and material (lagrangian) descriptions of the motion. Material derivative. Velocity and acceleration. Trajectories and streamlines.

1.2. Deformation and vorticity tensors. Physical interpretation.

1.3. Reynolds' transport theorem.

### 2. Fundamental equations in Continuum Mechanics.

2.1. Conservation of mass: continuity equation.

2.2. Long-range (volume) forces and contact (surface) forces. Stress tensor.

2.3. Momentum balance: equation of motion. Mechanical energy balance.

2.4. Angular momentum balance: symmetry of the stress tensor.

2.5. Conservation of energy and first principle of Thermodynamics.

2.6. Constitutive relations.

### 3. Viscous fluids. Navier-Stokes equation.

3.1. Hydrostatic pressure. Ideal fluid model. Euler's equation.

3.2. Stress tensor for a linearly viscous fluid. Newtonian fluid model. Coefficients of viscosity. Derivation of Navier-Stokes' equation.

3.3. Boundary conditions.

3.4. Scale analysis and dimensionless numbers.

### 4. Energy equation for a Newtonian fluid.

4.1. Second principle of Thermodynamics. Energy equation in the entropic representation. Concepts of adiabatic, isentropic and homoentropic motions.

4.2. Heat conduction. Entropy sources.

4.3. Alternative forms of expressing the energy equation.

### 5. Circulation and vorticity.

5.1. Circulation and vorticity. Vortex tubes. Some kinematic results.

5.2. Theorems of Kelvin and Helmholtz for ideal fluids.

5.3. Navier-Stokes' equation in terms of the vorticity. 2-D results.

5.4. Crocco's equation and Bernoulli's theorems.

## Part II. Special topics.

### 6. The hydrodynamic equations in conservation form.

6.1. Momentum equation in conservation form. The momentum density flux tensor.

6.2. Energy equation in conservation form. The energy density flux vector.

6.3. Derivation of the jump relations across a discontinuity. Tangential discontinuities and shock fronts. Rankine-Hugoniot relations.

### 7. Chandrasekhar's adiabatic exponents in compressible hydrodynamics.

7.1. Definition of Chandrasekhar's adiabatic exponents as material response functions.

7.2. Physical interpretation of the adiabatic exponents $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$.

7.3. Alternative representations of the energy equation in compressible hydrodynamics in terms of the material functions $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$.

### 8. The virial theorem and astrophysical applications.

8.1. Derivation of the scalar virial theorem in Hydrodynamics. Interpretation of the various terms.

8.2. Some astrophysical applications of the virial theorem: Stars in hydrostatic equilibrium; restriction on the ratio of specific heats $\gamma$. Quasistatic contraction as possible energy source. Kelvin-Helmholtz time scale. Free-fall time scale. Derivation of the relationship between the pulsation period and the mean stellar density for pulsating stars.