Exercises for Partial Differential Equations I

1. The one dimensional diffusion equation is given by:

$$\frac{\partial \rho(x,t)}{\partial t} = D \cdot \frac{\partial^2 \rho(x,t)}{\partial x^2}$$

Solve this equation by separation of variables
(Ansatz: $\rho(x,t) = \rho_1(t) \cdot \rho_2(x)$)
on a grid $0 \leq x \leq L_x$, $t \geq 0$ and $D > 0$
for the following boundary and initial conditions:

(a) Dirichlet boundary condition: $\rho(0,t) = \rho(L_x,t) = 0$.
Initial condition: $\rho(x,0) = A \cdot \exp(-\frac{(x-x_0)^2}{l})$

(b) Dirichlet boundary condition: $\rho(0,t) = \rho(L_x,t) = 0$.
Initial condition: $\rho(x,0) = A \,\text{for}\, x_0 \leq x \leq x_1$ and $\rho(x,0) = 0$ else.

(c) Dirichlet boundary condition: $\rho(0,t) = T_0,\ \rho(L_x,t) = T_1$
Initial condition: $\rho(x,0) = 0$ for $0 < x < L_x$

(d) Von Neumann boundary conditions: $\frac{\partial \rho}{\partial x}(0,t) = \frac{\partial \rho}{\partial x}(Lx,t) = 0$.
Initial conditions as in (a) and (b).

(e) Can you imagine how the final stationary state distribution
$\rho(x, t \to \infty)$ will look for cases a-d?

For an implementation in IDL you can start with the program
O : \wiegelmann\PDE_lecture\lecture_diffusion_draft.pro
Suggested paramters (feel free to use others):
$L_x = 10$, $D = 0.5$, $A = 2.0$, $l = 1.5$, $T_0 = 0.5$, $T_1 = 2.0$, $x_0 = 5.0$, $x_1 = 7.0$, $0 \leq t \leq 20.0$