Exercises for Partial Differential Equations II

1. Poisson-equation in 2D:
   Gauss law (in dimensionless form) is given by $\nabla \cdot \mathbf{E} = \rho$. With $\mathbf{E} = -\nabla \phi$ we derive a Poisson equation. Here we consider the problem in 2D: $(\mathbf{E} = E_x(x,y)\mathbf{e_x} + E_y(x,y)\mathbf{e_y})$ which leads to:

   $$-\frac{\partial^2 \phi(x,y)}{\partial x^2} - \frac{\partial^2 \phi(x,y)}{\partial y^2} = \rho(x,y)$$

   Solve this equation numerically with Dirichlet boundary conditions $\phi = 0$ on all boundaries. Compute the potential $\phi(x,y)$ and the electric field $\mathbf{E}(x,y)$.

   (a) Jacobi method.

   (b) Gauss-Seidel method.

   (c) Successive Overrelaxation (SOR-method).

   (d) Try to find the optimum relaxation factor w for the SOR-method.

   (e) Investigate how the methods scale with the grid resolution. Say $h = 0.4, 0.2, 0.1$.

   For an implementation in IDL you can start with the program
   O : \wiegelmann\PDE_lecture\lecture_poisson2d_draft.pro
   The program computes a distribution of electric charges $\rho(x,y)$ on a grid $L_x = 12$, $L_y = 10$ with a grid resolution $\Delta x = \Delta y = h = 0.4$