Exercises for Partial Differential Equations III

Time dependent problems $\rho = \rho(x,t)$ in flux conservative form:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial F(\rho)}{\partial x}$$

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1. Advection equation:

$$\frac{\partial \rho}{\partial t} = -v \cdot \frac{\partial \rho}{\partial x}, \quad (1)$$

   where v is a constant velocity.

   Write a numerical code to solve this equation with:

   (a) First order upwind scheme.

   (b) Leap-Frog method. (Without and with artivicial viscosity)

   (c) Lax-Wendroff scheme.

   (d) Try first an initial Gauss-profile $\rho(x,0)$ and make a square box-test (similar as used in Exersice I) after. For which initial condition do the numerical schemes work better?

2. Inviscid Burgers' equation:

$$\frac{\partial \rho}{\partial t} = -v\,\rho \cdot \frac{\partial \rho}{\partial x}, \quad (2)$$

   where v is a constant.

   (a) Which term is nonlinear in equation (2)?

   (b) Equation (2) is not in conservative form. Please try to write it in this form.

   (c) Solve equation (2) with the different numerical schemes developed for the advection equation. If you used maximal flexibility for writing the advection codes almost no changes are necessary. Use a Gauss profile as initial state for $\rho(x,0)$.

For an implementation in IDL you can start with the program
O : \wiegelmann\PDE_lecture\lecture_advection_draft.pro
The program solves the advection equation with the help of the Lax-method. The function ρ_0 computes as initial condition a Gauss-profile and ρ_1 a square box.