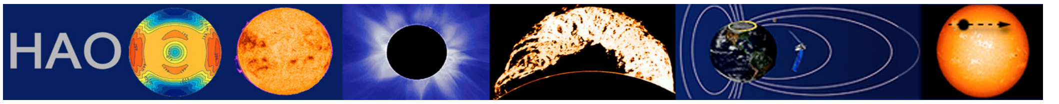


# Self-Similarity in Classical Fluids and MHD

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# 1-D Thermal Pulse

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial x^2} = 0$$

$$T(x,t) = \frac{T_0}{\sqrt{t}} \exp\left(-\frac{x^2}{4\kappa t}\right)$$

$$E = \int_{-\infty}^{\infty} T(x,t)\,dx = T_0 \int_{-\infty}^{\infty} \exp\left(-\frac{\zeta^2}{4\kappa}\right) d\zeta\,; \quad \zeta = \frac{x}{t^{1/2}}$$

$$T(x,t) = \frac{1}{\sqrt{t}} \int_{-\infty}^{\infty} \exp\left(-\frac{(x-x')^2}{4\kappa t}\right) T(x',0)\,dx'$$

# Reduction from pde to ode

$$\varsigma = \frac{x}{t^{1/2}} \quad \& \quad T(x,t) = \frac{1}{t^{1/2}}\Theta(\varsigma)$$

$$\kappa \frac{d^2\Theta}{d\varsigma^2} + \varsigma \frac{d\Theta}{d\varsigma} + \frac{1}{2}\Theta = 0$$

$$\Theta = \exp\left(-\frac{\varsigma^2}{4\kappa}\right)$$

# Spherically-Symmetric Flows

$$\rho\left(\frac{\partial \mathrm{v}}{\partial t} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial r}\right) = -\frac{\partial \mathrm{p}}{\partial r}$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho \mathrm{v}\right) = 0$$

$$\frac{\partial}{\partial t}\left(p\rho^{-\gamma}\right) + \mathrm{v}\frac{\partial}{\partial r}\left(p\rho^{-\gamma}\right) = 0$$

# Sedov-Taylor Point Explosion

- Initial state: $\mathrm{E}_0 \sim ML^2T^{-2}, \quad \rho_0 \sim ML^{-3}$
- Similarity variable:

$$\zeta = r\left(\frac{\rho_0}{\mathrm{E}_0 t^2}\right)^{\frac{1}{5}}$$

- The solution:

$$\mathrm{v} = \frac{4}{5(\gamma+1)}\frac{r}{t}\mathrm{V}(\zeta)$$

$$\rho = \rho_0 \frac{\gamma+1}{\gamma-1} D(\zeta)$$

$$p = \frac{8\rho_0}{25(\gamma+1)}\frac{r^2}{t^2}P(\zeta)$$

# A Realistic "Point" Explosion

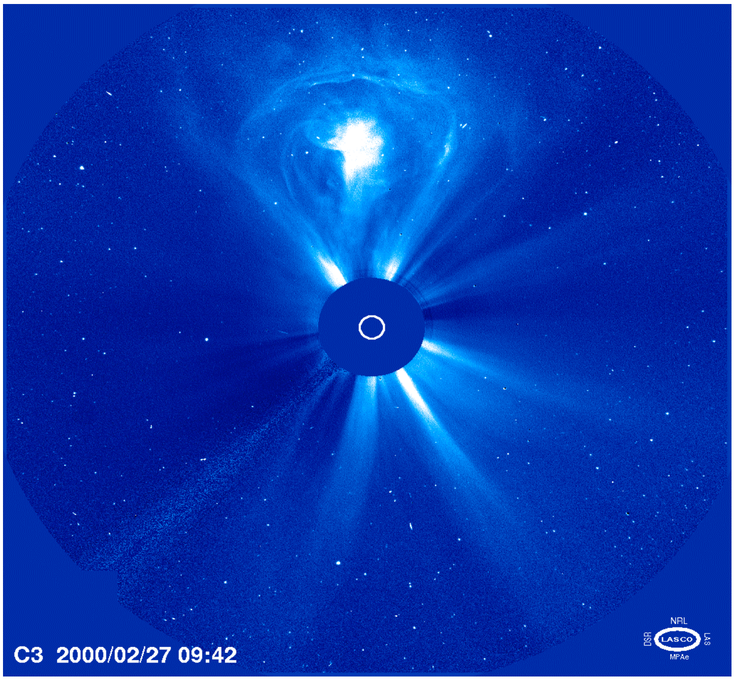
- Energy released: $\mathrm{E}_0$
- Fireball radius: $\mathrm{R}_0$
- Warm medium: $p_0, \rho_0$
- Length scales: $r_1 = \mathrm{R}_0, \quad r_2 = \left(\frac{\mathrm{E}_0}{\mathrm{p}_0}\right)^{1/3}$
- Time scales: $t_1 = \left(\rho_0 \mathrm{R}_0^5 / \mathrm{E}_0\right)^{1/2}, \quad t_2 = \left(\rho_0 \mathrm{E}_0^{2/3} / p_0^{5/3}\right)^{1/2}$
- Similarity regime:

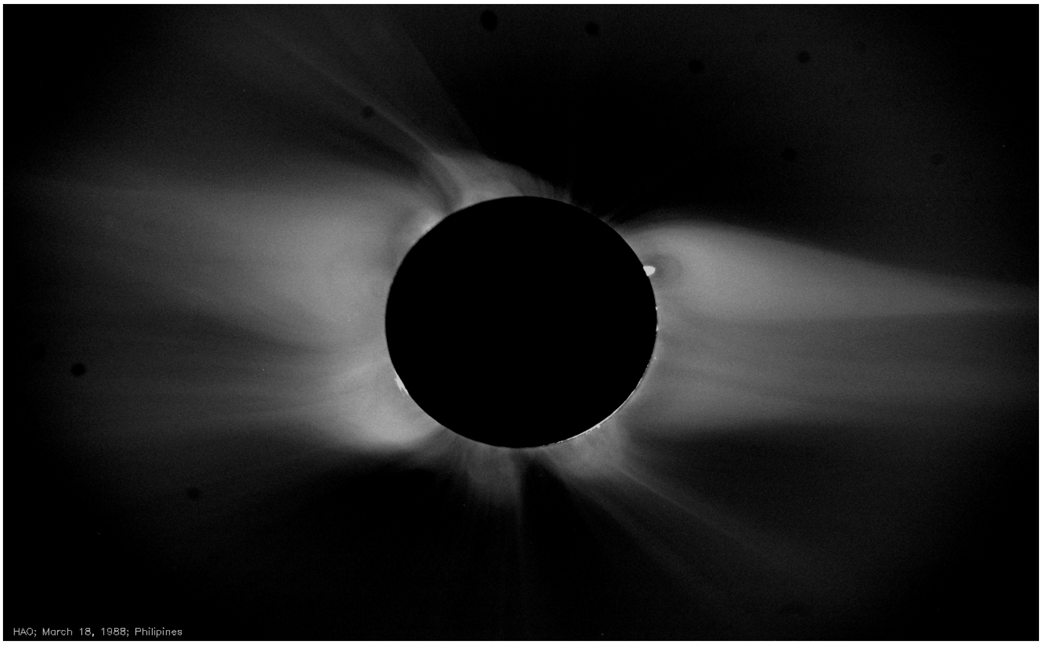
$$r_1 << r << r_2 \,, \quad t_1 << t << t_2$$

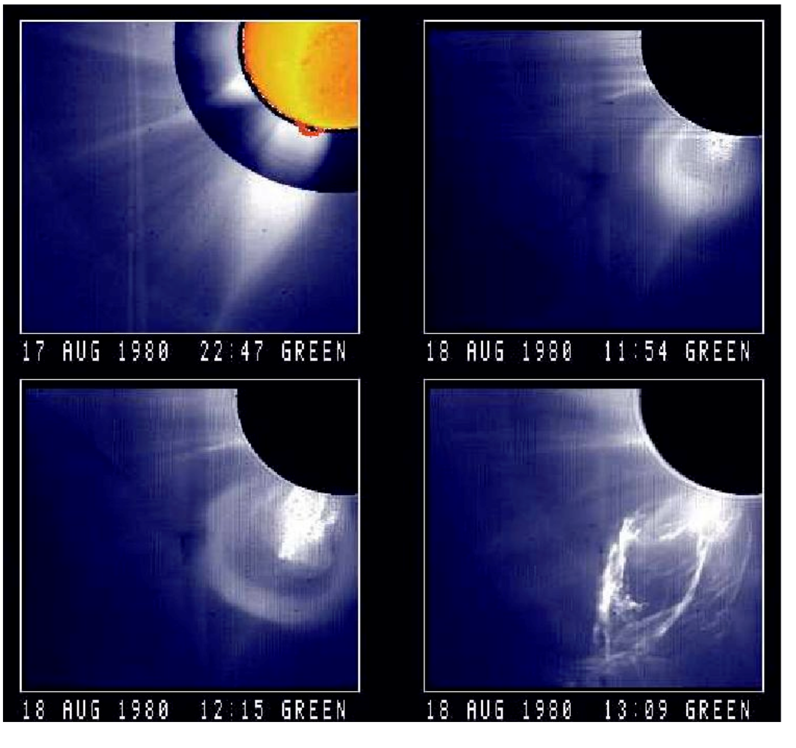
# Intermediate Asymptotics

Barenblatt & Zel'dovich (1972, Ann.Rev. Fluid Mech. 4, 285)

- Self-similar solutions are the intermediate asymptotic forms of a solution in a local space time region away from boundaries and its initial state.
- Self-similar solutions often turn out to be the "preferred" end state of a diversity of possible evolutions originating from very different initial states.




HAO; March 18, 1988; Philipines


17 AUG 1980 22:47 GREEN
18 AUG 1980 11:54 GREEN
18 AUG 1980 12:15 GREEN
18 AUG 1980 13:09 GREEN

# CME Kinetic Properties

- Typical CME mass ~ $5\times10^{15}\ g$
- Median Speed ~ $450\ km/s$
- Range ~ $50-2\times10^{3}\ km/s$
- G-escape Speed ~ $550\ km/s$
- Coronal Sound Speed ~ $120\ km/s$
- Coronal Alfven Speed ~ $1000\ km/s$

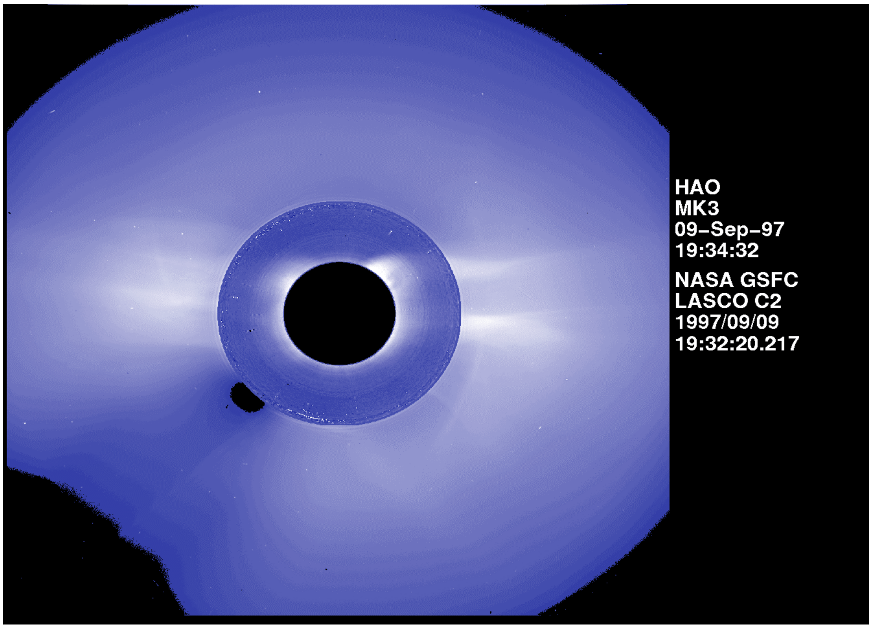
# Time-Dependent Ideal MHD

$$\rho\frac{d\vec{v}}{dt}=\frac{1}{4\pi}(\nabla\times\vec{B})\times\vec{B}-\nabla p-\rho\frac{GM_{Sun}}{r^2}\hat{r}$$

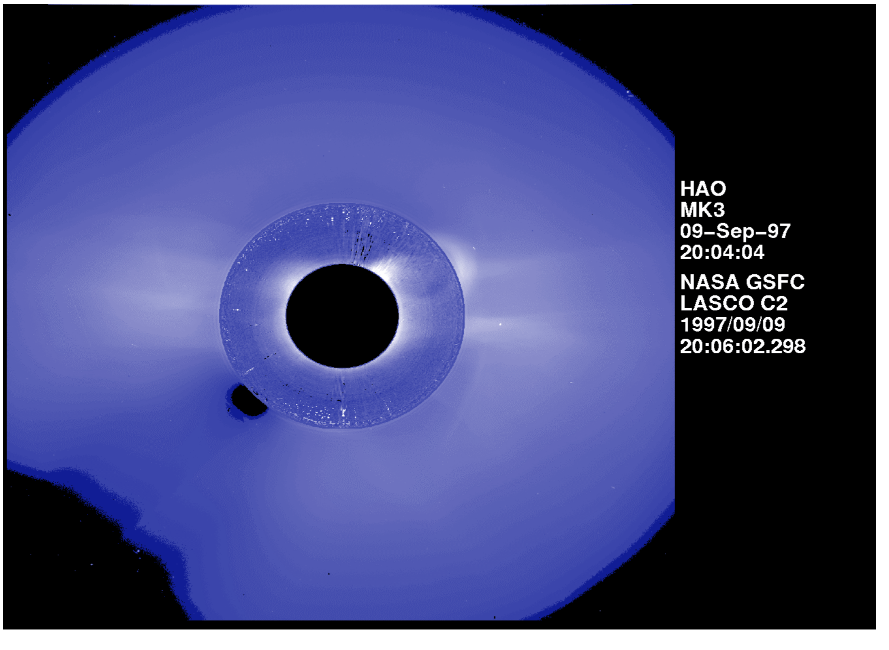
$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0$$

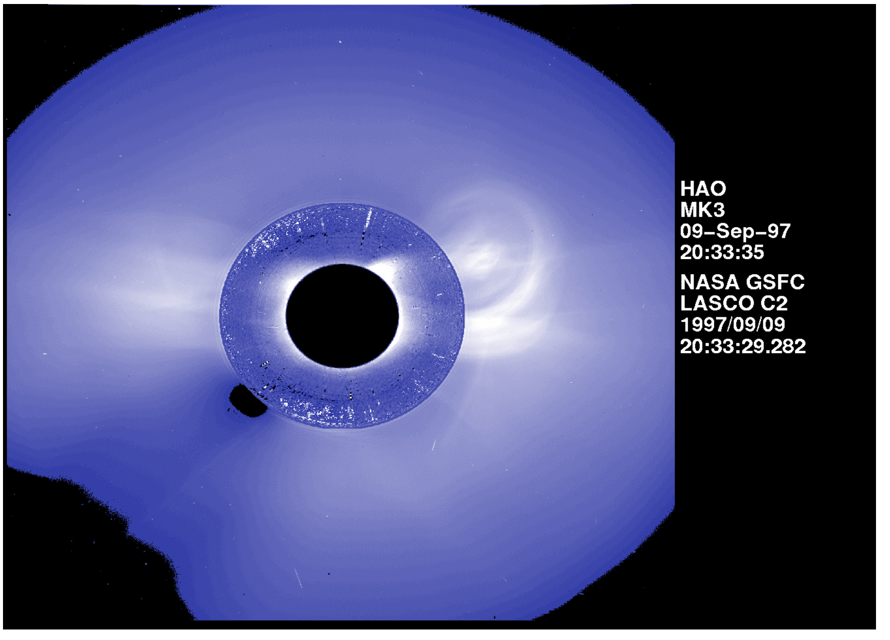
$$\frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{v}\times\vec{B})$$

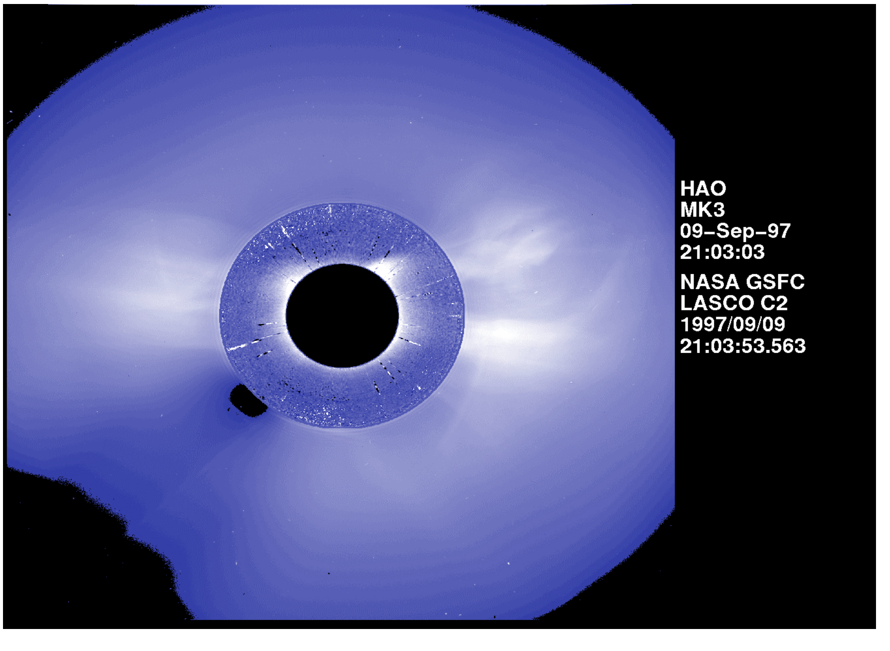
$$\frac{d}{dt}(p\rho^{-\gamma})=0$$

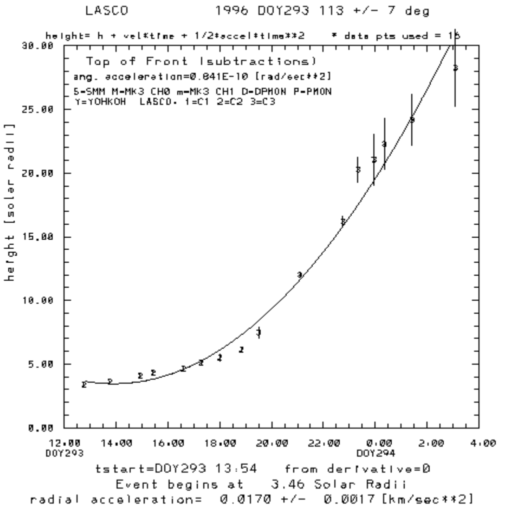


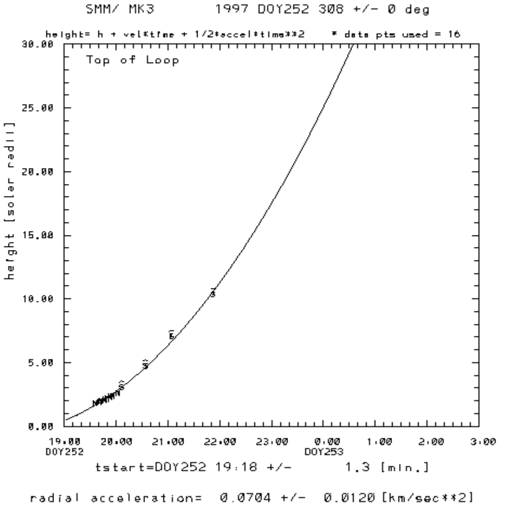

Darnell & Burkepile 2002

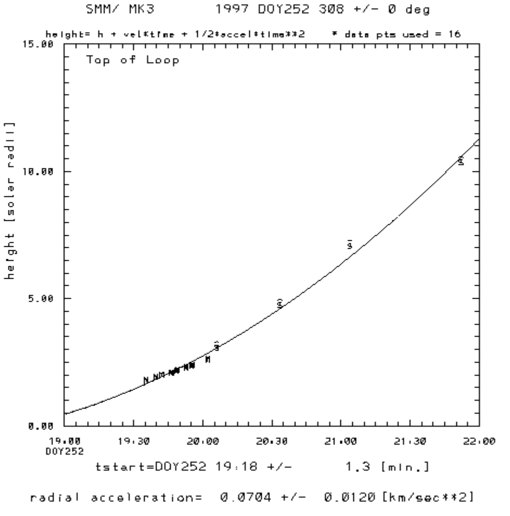



HAO
MK3
09–Sep–97
20:33:35
NASA GSFC
LASCO C2
1997/09/09
20:33:29.282


HAO
MK3
09–Sep–97
21:03:03
NASA GSFC
LASCO C2
1997/09/09
21:03:53.563


LASCO 1996 DOY293 113 +/- 7 deg
height= h + vel*time + 1/2*accel*time**2 * data pts used = 16
Top of Front (subtractions)
ang. acceleration=0.841E-10 [rad/sec**2]
S=SMM M=MK3 CH0 m=MK3 CH1 D=DPMON P=PMON
Y=YOHKOH LASCO: 1=C1 2=C2 3=C3
height [solar radii]
30.00
25.00
20.00
15.00
10.00
5.00
0.00
12:00
DOY293
14:00
16:00
18:00
20:00
22:00
0:00
DOY294
2:00
4:00
tstart=DOY293 13:54 from derivative=0
Event begins at 3.46 Solar Radii
radial acceleration= 0.0170 +/- 0.0017 [km/sec**2]


SMM/ MK3 1997 DOY252 308 +/- 0 deg
height= h + vel*time + 1/2*accel*time**2 * data pts used = 16
Top of Loop
height [solar radii]
30.00
25.00
20.00
15.00
10.00
5.00
0.00
19:00
DOY252
20:00
21:00
22:00
23:00
0:00
DOY253
1:00
2:00
3:00
tstart=DOY252 19:18 +/- 1.3 [min.]
radial acceleration= 0.0704 +/- 0.0120 [km/sec**2]




# Homologous Expansion

$$\phi \sim R^{-1}$$

$$\rho \sim R^{-3}; \qquad \rho\phi \sim R^{-4}$$

$$B \sim R^{-2}; \qquad B^2 \sim R^{-4}$$

$$p \sim \rho^{\lambda} \sim R^{-3\gamma};$$

$$\textit{if } \gamma = 4/3, \quad \textit{then } \; p \sim R^{-4}$$

# Time-Dependent Ideal MHD

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}.\nabla\right)\vec{v}\right)=\frac{1}{4\pi}(\nabla\times\vec{B})\times\vec{B}-\nabla p-\rho\frac{GM_{Sun}}{r^2}\hat{r}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0$$

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{v}\times\vec{B})$$

$$\frac{d}{dt}(p\rho^{-4/3})=0$$

## Homologous Radial Flow: Zero-Force Flow

$$\mathbf{v}=\frac{r}{t}\hat{\mathbf{r}}$$

$$\frac{\partial\mathbf{v}}{\partial t}+\left(\mathbf{v}.\nabla\right)\mathbf{v}=0$$

$$\frac{dr}{dt}=\frac{r}{t}\Rightarrow\frac{d}{dt}\left(\frac{r}{t}\right)=0$$

## Homologous Radial Flow: Mass Conservation

$$\mathbf{v} = \frac{r}{t}\hat{\mathbf{r}}$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho \, \mathrm{v_r}\right) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^3}{t}\rho\right) = 0 \Rightarrow \rho = \frac{1}{t^3}\mathrm{D}(\frac{\mathrm{r}}{\mathrm{t}},\theta,\phi)$$

## A Simple Class of Self-Similar MHD Solutions

$$\mathbf{v} = \frac{r}{t}\hat{\mathbf{r}}, \quad \zeta = \frac{r}{t}$$

$$\rho = \frac{1}{t^3}\mathrm{D}(\zeta,\theta,\phi) = \frac{\zeta^3}{r^3}\mathrm{D}(\zeta,\theta,\phi)$$

$$p = \frac{1}{t^4}\mathrm{P}(\zeta,\theta,\phi) = \frac{\zeta^4}{r^4}\mathrm{P}(\zeta,\theta,\phi)$$

$$\vec{\mathrm{B}} = \frac{1}{t^2}\vec{\mathrm{H}}(\zeta,\theta,\phi) = \frac{\zeta^2}{r^2}\vec{\mathrm{H}}(\zeta,\theta,\phi)$$

$$\frac{1}{4\pi}(\nabla^* \times \vec{\mathrm{H}}) \times \vec{\mathrm{H}} - \nabla^* P - \mathrm{D}\frac{\mathrm{GM_{Sun}}}{\zeta^2}\hat{\mathrm{r}} \quad = 0$$

## A More Sophisticated Class of Self-Similar MHD Solutions

$$\mathbf{v} = \frac{r}{\Phi(t)}\frac{d\Phi}{dt}\hat{\mathbf{r}}, \quad \zeta = \frac{r}{\Phi(t)}$$

$$\rho = \frac{1}{\Phi^3}\mathrm{D}(\zeta,\theta,\phi)$$

$$p = \frac{1}{\Phi^4}\mathrm{P}(\zeta,\theta,\phi)$$

$$\vec{\mathrm{B}} = \frac{1}{\Phi^2}\vec{\mathrm{H}}(\zeta,\theta,\phi)$$

$$\frac{1}{4\pi}(\nabla * \times \vec{\mathrm{H}}) \times \vec{\mathrm{H}} - \nabla * P - \mathrm{D}\left(\frac{\mathrm{GM_{Sun}}}{\zeta^2} + \alpha\zeta\right)\hat{\mathrm{r}} = 0$$

$$\frac{\mathrm{d}^2\Phi}{dt^2} = \frac{\alpha}{\Phi^2}$$

# A Non-Galilean Transformation

- A non-Galiliean transformation introduces inertial forces, e.g., the Coriolis force in a uniformly rotating frame of reference.
- The similarity space $\left(\zeta = \frac{r}{\Phi(t)}, \theta, \phi\right)$ introduces a pure radial force of magnitude $\alpha\zeta$ - the net force driving the flow.

# Stability Problems in Similarity Space

- With no loss of generality:

$$(r,\theta,\phi,t\} \rightarrow (\zeta = \frac{t}{\Phi(t)}, \theta, \phi, t)$$

- Time-dependent linear perturbations about a static state in $(\zeta, \theta, \phi)$ space – a classic stability problem.

## A More Sophisticated Class of Self-Similar MHD Solutions

$$\mathbf{v} = \frac{r}{\Phi(t)}\frac{d\Phi}{dt}\hat{\mathbf{r}}, \quad \zeta = \frac{r}{\Phi(t)}$$

$$\rho = \frac{1}{\Phi^3}\mathrm{D}(\zeta,\theta,\phi)$$

$$p = \frac{1}{\Phi^4}\mathrm{P}(\zeta,\theta,\phi)$$

$$\vec{\mathrm{B}} = \frac{1}{\Phi^2}\vec{\mathrm{H}}(\zeta,\theta,\phi)$$

$$\frac{1}{4\pi}(\nabla * \times \vec{\mathrm{H}}) \times \vec{\mathrm{H}} - \nabla * P - \mathrm{D}\left(\frac{\mathrm{GM_{Sun}}}{\zeta^2} + \alpha\zeta\right)\hat{\mathrm{r}} = 0$$

$$\frac{\mathrm{d}^2\Phi}{dt^2} = \frac{\alpha}{\Phi^2}$$

# Velocity vs Distance



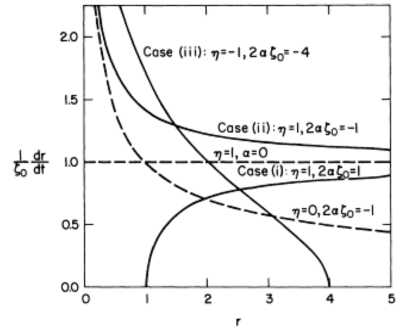



FIG. 1.—Lagrangian velocity profiles

parameter $\alpha$ determines whether there is a positive or negative transfer of momentum into kinetic motion, as we pointed out in connection with equation (28). Thus,

ing relationship to each other. The *relative* magnitudes of the various forces, including the rate of change of momentum, are unchanging in time. This is expressed by equation (27) in $\zeta$-$\theta$ space. What changes is that the *absolute* magnitudes of these forces all decrease with time proportionately, according to the scaling with time given by equations (19), (23), (16), and (17). In this regard, it is interesting to note that the marginal case $\eta = 0$ corresponds to the escape of plasma and magnetic field to infinity with the velocity tending to zero. In a strict sense, matter is escaping the bound of gravity, except that it does so with no net gain of kinetic energy.

The constant terminal velocity implies that, if the self-similar solution is valid out to the orbit of the Earth, a localized density enhancement of the transient would be observed at 1 AU to have a constant Lagrangian velocity less than the expected MHD shock speed. This enhancement should be distinguished from the usual interplanetary shock (Parker 1963). Such density enhancements, moving at nearly the background solar wind speed, have been observed by satellite, and Gosling *et al.* (1977) suggested that they are transient-related. Our self-similar solutions for $\eta = V_0^2 > 0$ provide a theoretical basis for this suggestion.

# A More Sophisticated Class of Self-Similar MHD Solutions

$$\mathbf{v} = \frac{r}{\Phi(t)}\frac{d\Phi}{dt}\hat{\mathbf{r}}, \quad \zeta = \frac{r}{\Phi(t)}$$

$$\rho = \frac{1}{\Phi^3}\mathrm{D}(\zeta,\theta,\phi)$$

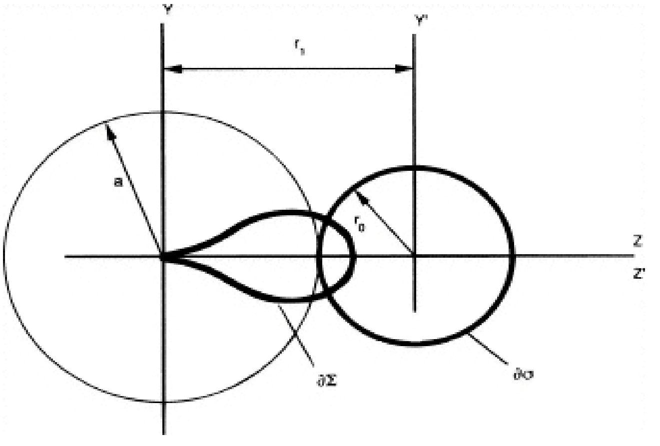
$$p = \frac{1}{\Phi^4}\mathrm{P}(\zeta,\theta,\phi)$$

$$\vec{\mathrm{B}} = \frac{1}{\Phi^2}\vec{\mathrm{H}}(\zeta,\theta,\phi)$$

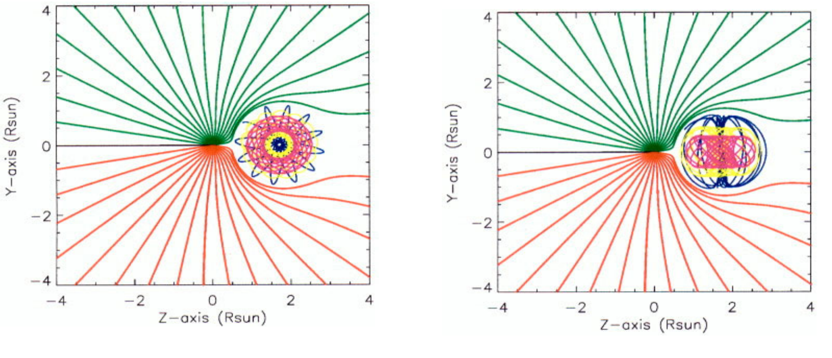
$$\frac{1}{4\pi}(\nabla^* \times \vec{\mathrm{H}}) \times \vec{\mathrm{H}} - \nabla^* P - \mathrm{D}\left(\frac{\mathrm{GM_{Sun}}}{\zeta^2} + \alpha\zeta\right)\hat{\mathrm{r}} = 0$$

$$\frac{\mathrm{d}^2\Phi}{dt^2} = \frac{\alpha}{\Phi^2}$$

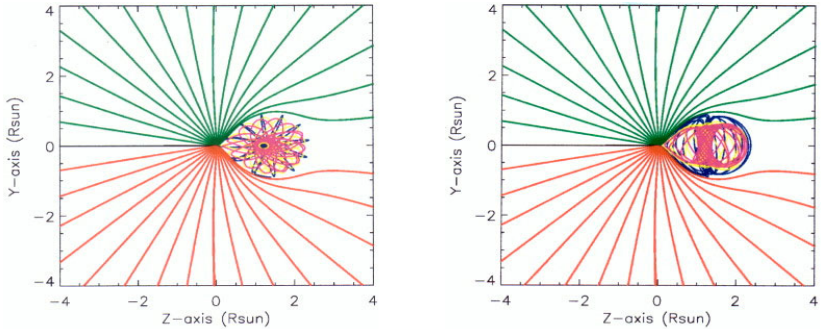
### Squeezing a 3D Solution out of a 2D Solution



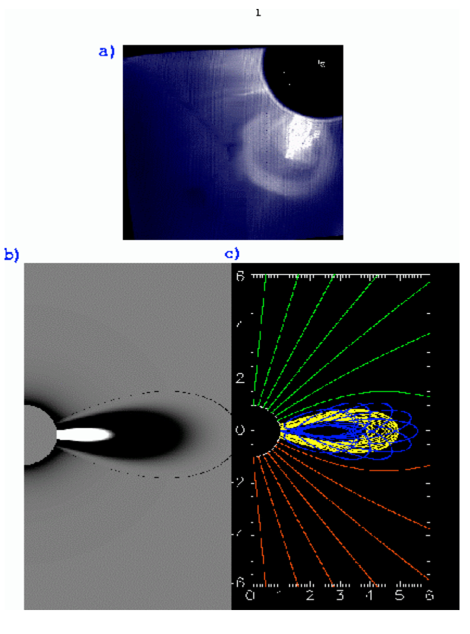

# Unsqueezed Solution



# Squeezed Solution




# A Theoretical 3-Part CME




Gibson & Low (1998, 2000)

# Summary

- Similarity solutions as long-time end-states.
- Compatible scaling laws of the thermal energy of a polytropic gas of 4/3 index with gravitational and magnetic energy.
- Observed broad range of CME speeds.
- The 3-part structure of CMEs has its origin in the initial state.
- Do similarity flows occur in CMEs?

# Movies from Gibson & Low

- Frot~1 shows a time sequence of the magnetic field of the CME making its way out surrounded by the global open magnetic field of the corona.
- Sigmoid shows the 3 D magnetic field of the CME from different perspective.
- Pb3 shows a time sequence of the CME in simulated Thomson-scattered light.