IMPRS
Solar System Seminar
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# Why does the magnetic field of the Earth reverse?

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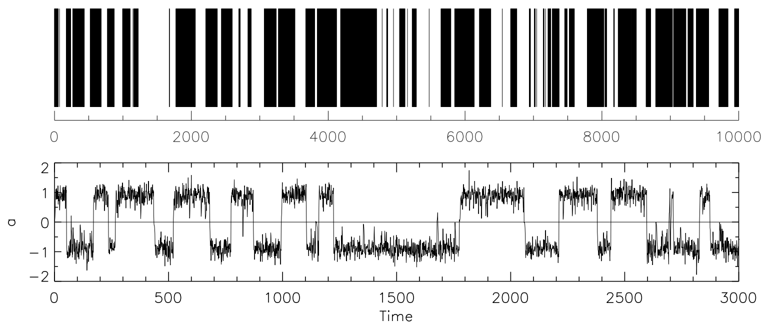
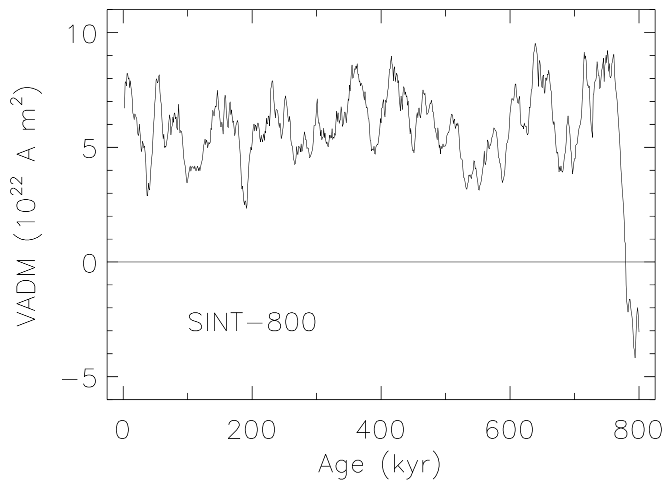
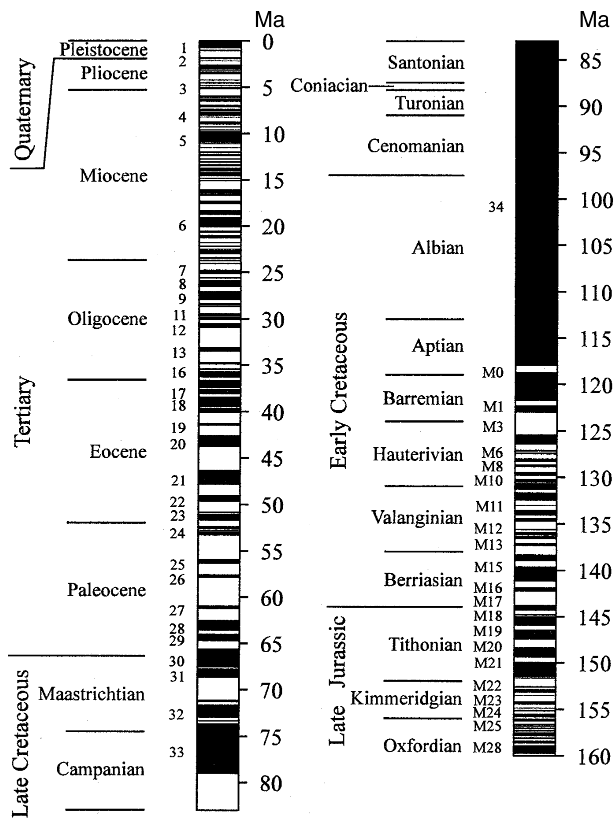



## 1. Geomagnetic field

- >99% of matter in universe is plasma: gas of electrons, ions and neutral particles
- moving charged particles → electric currents → magnetic fields
- basic process → magnetic fields everywhere in cosmos: planets, stars, accretion disks, galaxies
- magnetic field of Earth oldest known magnetic field of a celestial body: compass, Gilbert (1600)
- Gauss (1838): spherical harmonics, sources in interior, mainly dipolar, $B_E \approx 0.3\,\mathrm{G}$, $B_P \approx 0.6\,\mathrm{G}$, tilt $\approx 11^\circ$, offset $\approx$ 450 km $\approx 0.07\,R_E$, $B_D \sim r^{-3}$
- higher multipoles superposed
- secular variation
  - westdrift of nonaxisymmetric components of $0.18^\circ$/yr
  - pole wobble
  - intensity variations
- polarity reversals, $10^3$ yr $\leftrightarrow$ $10^{5\ldots7}$ yr

SINT-800 VADM (Guyodo & Valet, 1999)



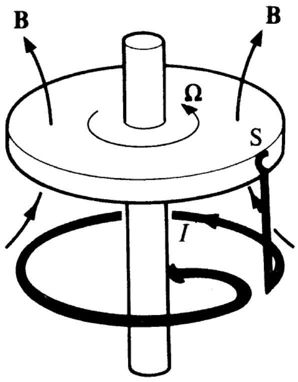



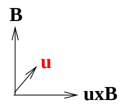
## 2. Dynamo theory basics

- composition of Earth
  - solid inner core, $r \approx$ 1200 km, Fe, Ni
  - fluid outer core, $r \approx$ 3500 km, good electrical conductor, thermal and chemical convection, location of electric currents, which excite the geomagnetic field
  - mantle, poorly conducting; thin crust
- decay time in core: $10^{4...5}$ yr
- electromagnetic induction:

  $\boldsymbol{u} \times \boldsymbol{B} \rightarrow \boldsymbol{j} \rightarrow \boldsymbol{B}\ (\rightarrow \boldsymbol{u})$

  self-excitation, dynamo-electrical principle (Siemens 1867)
- disk dynamo:

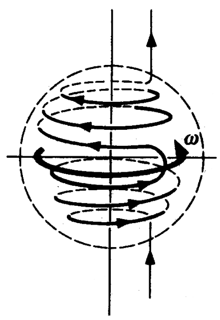


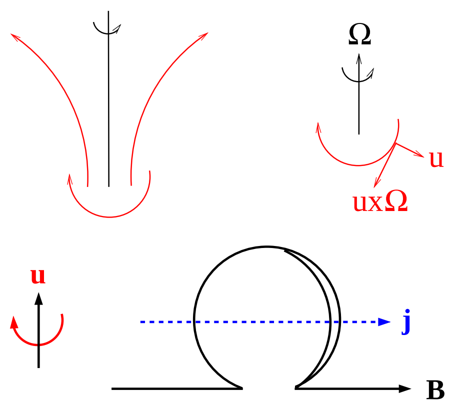

- homogenous dynamo, complicated flows, induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \text{rot}\,(\boldsymbol{u} \times \boldsymbol{B} - \eta\, \text{rot}\boldsymbol{B})\ , \quad \boldsymbol{B} = \boldsymbol{B}_{\text{pol}} + \boldsymbol{B}_{\text{tor}}$$

- differential rotation




- helical convection (Parker 1955)



- theory of mean fields (Steenbeck, Krause & Rädler 1966)

  $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{B}'$ ,  $\boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}'$

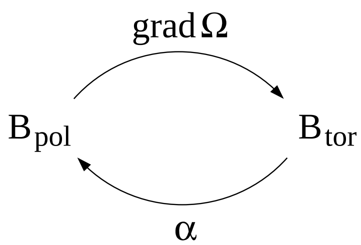
  average: ensemble, azimutal, spatial, temporal

  $$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \mathrm{rot}\,(\overline{\boldsymbol{u}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{u}' \times \boldsymbol{B}'} - \eta\,\mathrm{rot}\overline{\boldsymbol{B}})$$

  $$\overline{\boldsymbol{u}' \times \boldsymbol{B}'} = \alpha \overline{\boldsymbol{B}} - \eta_T\,\mathrm{rot}\overline{\boldsymbol{B}}$$

  $$\alpha = -\frac{1}{3}\int \overline{\boldsymbol{u}'(t)\cdot \mathrm{rot}\,\boldsymbol{u}'(t-\tau)}\,d\tau\ , \quad \eta_T = \frac{1}{3}\int \overline{\boldsymbol{u}'(t)\cdot \boldsymbol{u}'(t-\tau)}\,d\tau \gg \eta$$

- $\alpha\Omega$-dynamo




- dynamo number $C = R_\alpha R_\Omega = \alpha_0 R/\eta_T \cdot \Omega_0' R^3/\eta_T$

- stochastic helicity fluctuations $\delta\alpha$ of each convective cell

  $$\alpha = \alpha_0(\theta) + \delta\alpha\,(\theta, t) \quad \text{with} \quad \frac{\delta\alpha}{\alpha_0} = \frac{f_{\mathrm{rms}} F(\theta, t)}{\sqrt{2N_c \sin\theta}}$$

  correlation length $R\theta_c = R\pi/N_c$ , correlation time $\tau_c$
  forcing parameter $f_{\mathrm{rms}}^2 \theta_c^2 \tau_c$

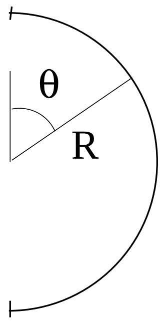
- back reaction of magnetic field on velocity ($\overline{\boldsymbol{u}}$ , $\boldsymbol{u}'$)
  through Lorentz force, e.g. $\alpha(B)$-quenching

- sign of $\boldsymbol{B}$ free

## 3. 1D mean-field $\alpha\Omega$-dynamo model

- azimuthal averages, $\boldsymbol{B}$ axisym. component of field

  $\boldsymbol{B} = \boldsymbol{B}(\theta, t) \exp(ikR)$ ,  $\boldsymbol{B} = \mathrm{rot}(P\boldsymbol{e}_\phi) + T\boldsymbol{e}_\phi$

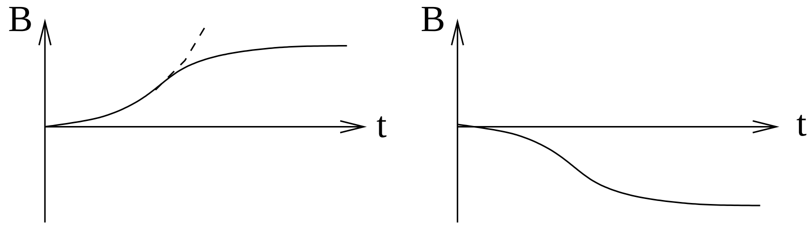



- dimensionless $\alpha\Omega$-dynamo equation

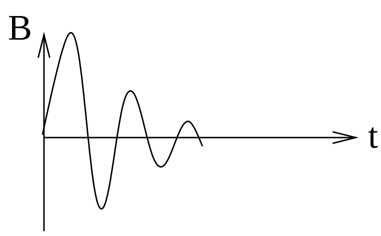
$$\frac{\partial}{\partial t}\begin{pmatrix} P \\ T \end{pmatrix} = \begin{pmatrix} \Delta & \alpha/\alpha_0 \\ C(\partial/\partial\theta)\sin\theta & \Delta \end{pmatrix}\begin{pmatrix} P \\ T \end{pmatrix}$$

  diffusion operator $\Delta$
  dynamo number $C$
  $\alpha$ fluctuations and quenching: $\alpha = \alpha_0(\theta, B) + \delta\alpha\,(\theta, t)$

- mode structure:
  - fundamental mode: monotonous dipole, supercritical, nonlinearly saturated

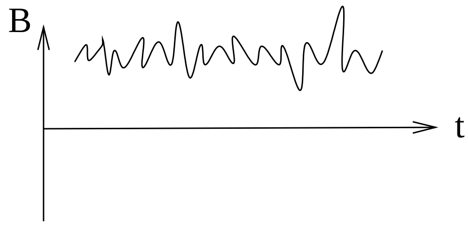



  - overtones: dipolar and quadrupolar, oscillatory, decaying

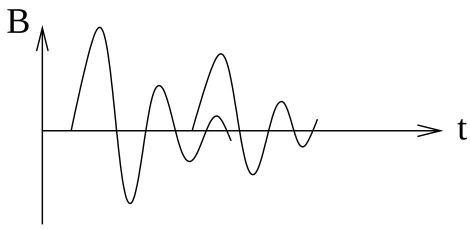


- effect of $\alpha$-fluctuations $\delta\alpha$:

  $\longrightarrow$ variation of fundamental mode amplitude

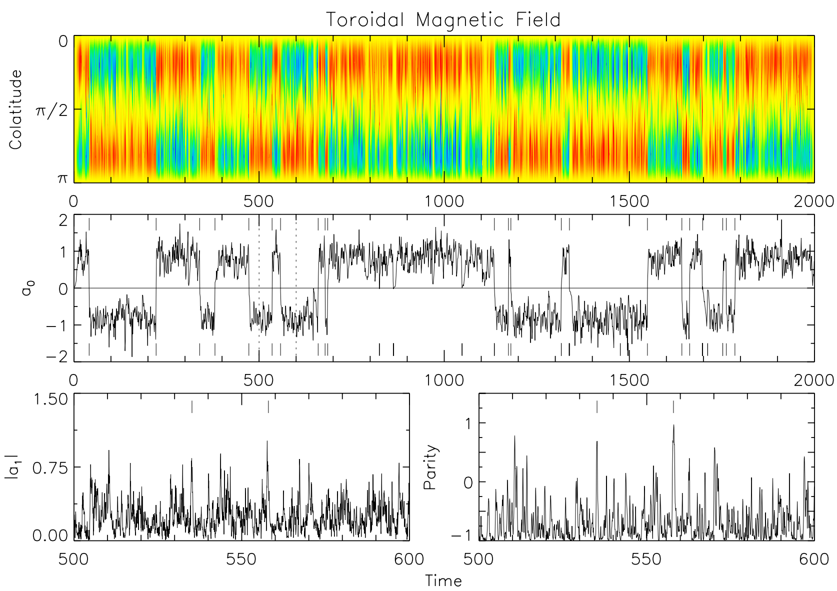



  $\longrightarrow$ stochastic excitation of higher modes



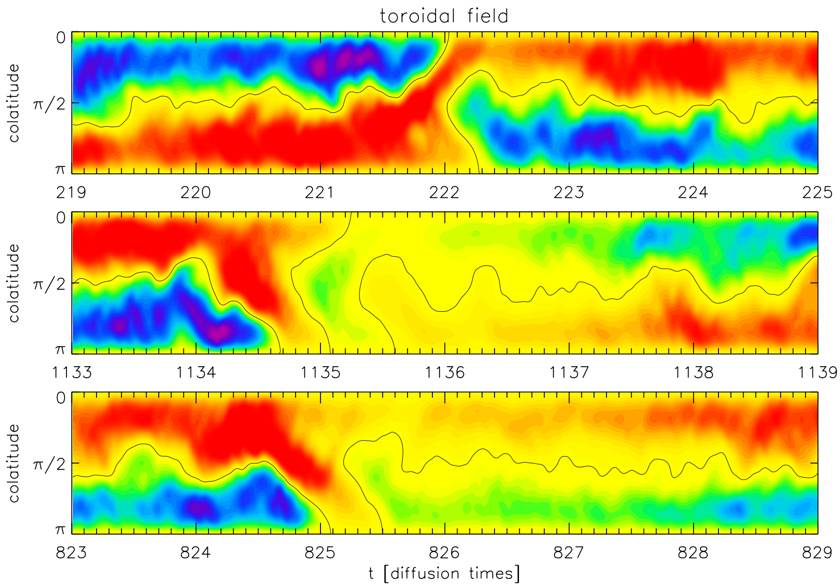

## 4. Numerical results

- standard case: $kR = 0.5,\ C_{\rm crit} = 57.9,\ C = 100,$
  $f_{\rm rms} = 6.4,\ \theta_c = \pi/10,\ \tau_c = 5\cdot 10^{-2}$




  – part of much longer run with $\sim$ 500 reversals
  – secular dipole ampl. variation $dv = \langle(a - \langle a\rangle)^2\rangle/\langle a\rangle^2 \approx 0.16$
  – rapid reversals with $T_r \approx 100\,\tau_D$

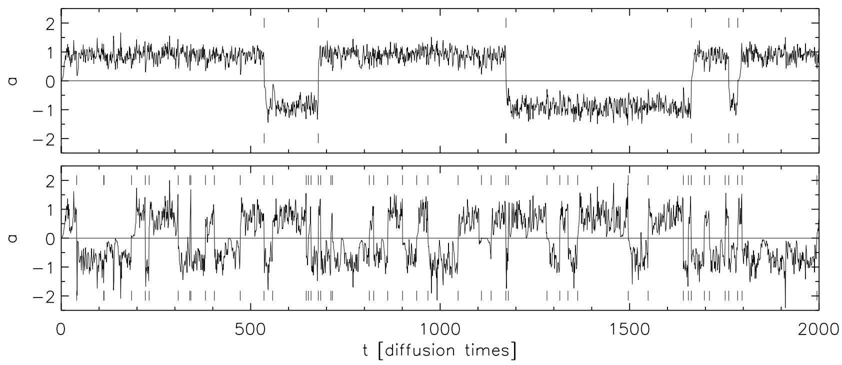
- reversals through decay of fundamental mode and short-time dominance of oscillating overtones




- variation of stochastic forcing:

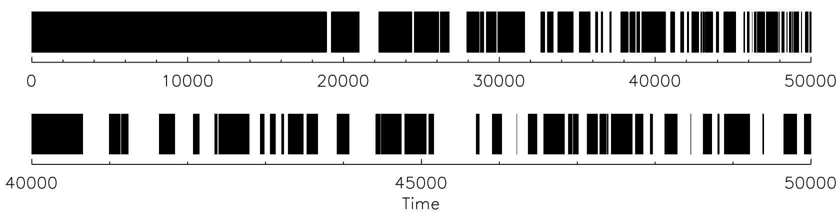
top: $f_{\rm rms} = 5.4$ $\curvearrowright$ $dv \approx 0.07,\ T_r \approx 500\,\tau_D$

bottom: $f_{\rm rms} = 7.4$ $\curvearrowright$ $dv \approx 0.3,\ T_r \approx 35\,\tau_D$

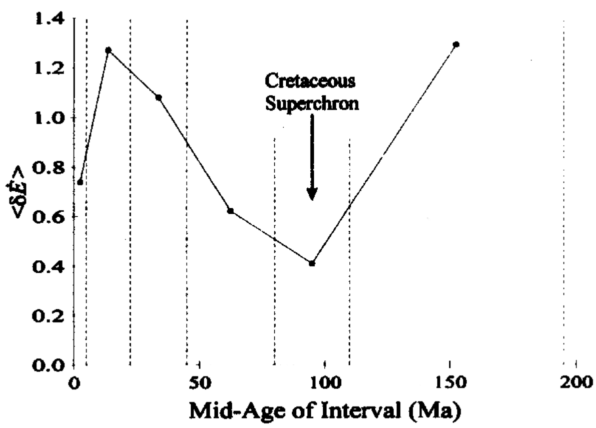



- linear increase of forcing from $f_{\rm rms} = 4.0 \ldots 6.5$

$\curvearrowright$ $dv \approx 0.03 \ldots 0.17$ and $T_r \approx 20000 \ldots 90\,\tau_D$



- variability / secular variation ↔ reversal rate




## 5. Theoretical analysis

- Fokker-Planck equation:
  - expansion of $\boldsymbol{B}$ into unperturbed eigenmodes $\boldsymbol{b}_i$

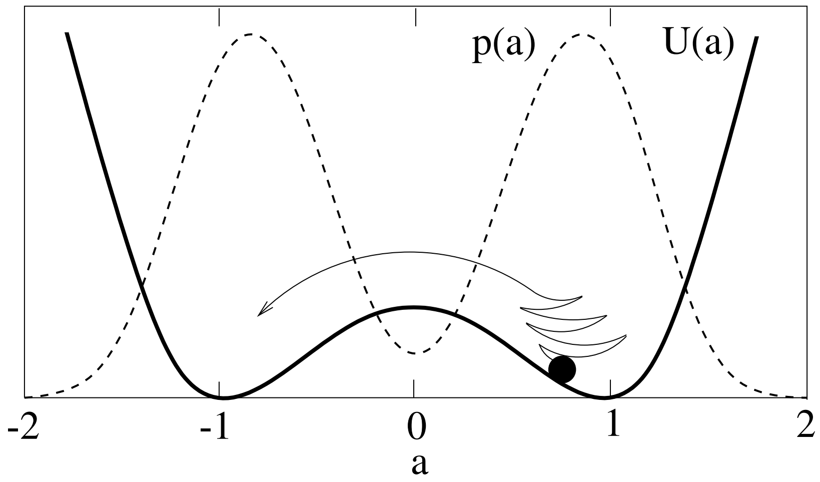
    $$\boldsymbol{B}(\theta, t) = \sum_i a_i(t)\boldsymbol{b}_i(\theta)$$

    $$\frac{\partial a_i}{\partial t} = \lambda_i a_i + (1 - a_0^2)\sum_k N_{ik} a_k + \sum_k F_{ik}(t) a_k$$
  - averaging over fluctuations

    $$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial a} S a + \frac{1}{2}\frac{\partial^2}{\partial a^2} D p$$

    $p(a)$ probability distribution of fundamental dipole amplitude $a = a_0$
  - drift term $S = \Lambda(1 - a^2)a = -dU/da$
    determined by dynamo model (supercriticality, nonlinearity)
  - diffusion term $D = D_0 a^2 + D_1(a),\ D_1(0) \neq 0$
    depends on forcing $f_{\mathrm{rms}}^2 \theta_c^2 \tau_c$
    variation of fundamental mode $(F_{00}, D_0)$
    influence of overtones $(F_{0k}, D_1)$

- bistable oscillator

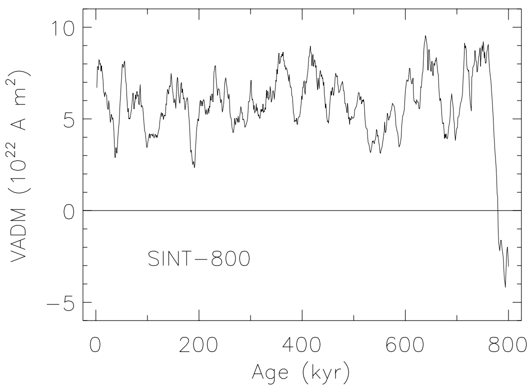



  - $U(a)$ given by $-\partial U/\partial a = S$
  - symmetry, central hill, side walls, wells
  - reversals through overtones ($D_1$)
  - reversals fast

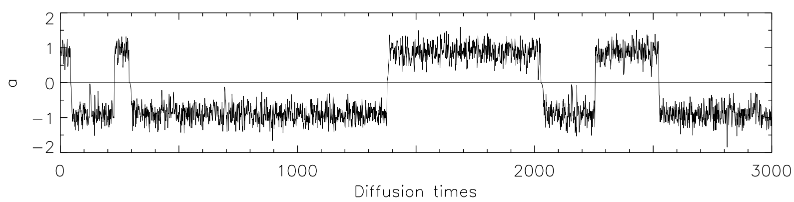
- amplitude distribution $p(a) \propto D^{-1} \exp \int_0^a 2SD^{-1}dx \quad \curvearrowright \quad dv,\ T_r$
- $f_{\mathrm{rms}} \nearrow \quad \curvearrowright \quad dv \nearrow \quad \curvearrowright \quad T_r \downarrow$
- ball in honey, reversal is random process, no specific cause

## 6. Comparison with SINT-800 data

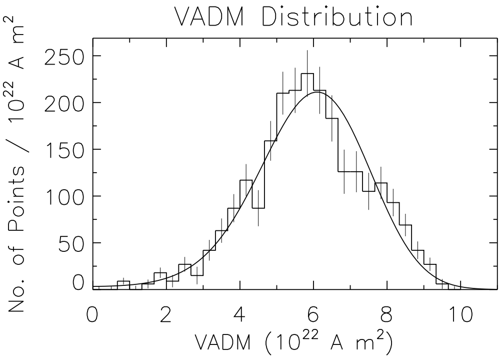
- SINT-800 VADM



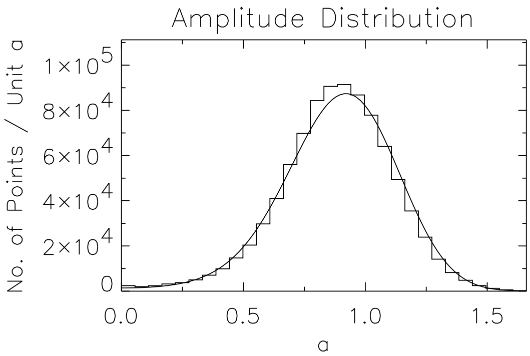

- secular VADM variation $dv = 0.071 \pm 0.003$
- dynamo model $\curvearrowright$ $T_r \approx 500\tau_D$



• VADM distribution




• comparison with dynamo

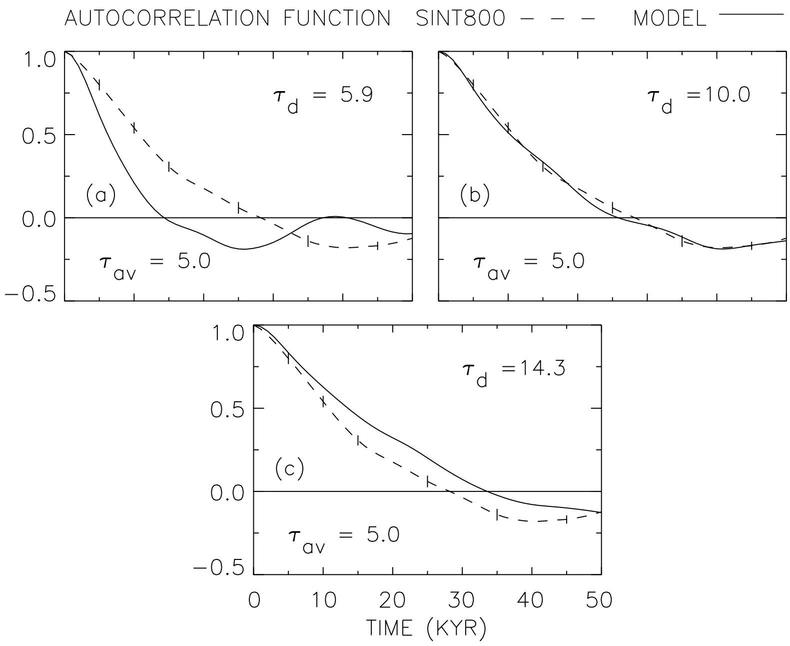



## 7. Diffusion time scale $\tau_D$

• VADM($t$) and $a(t)$ are essentially the same

• absolute time ↔ turbulent diffusion time

• adjust $\tau_D$ until autocorrelation functions match




• $\tau_D \approx 10^4$ yr